Inflation Dynamics under Fiscal Deficit Regime-Switching in Mexico

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Abstract

We investigate the role of the specification of the money-demand function within the theory of inflation dynamics. To that end, we extend the model of Sargent, Williams, and Zha (2009) to incorporate Selden-Latané's money-demand function. Our model exhibits new equilibrium dynamics and therefore new policy implications about what have been the best reforms to accomplish and maintain low and stable inflation. Our model also predicts a tighter historical relationship between fiscal and monetary policy; thus, we establish a stronger case for fiscal dominance as being the origin of Mexico's inflation instability during the last three decades of the twentieth century. We argue that avoiding Cagan's paradox is crucial for studying the relationship between fiscal and monetary policy. We estimate our model and explore the dynamics of inflation, inflation expectations, and seigniorage-financed fiscal deficits in Mexico, along with the changes in the deficit's mean and volatility regimes.

Keywords: Inflation, Inflation Expectations, Public Deficit, Fiscal Deficit, Regime-Switching, Monetary Policy, Fiscal Policy.

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1 Introduction

In Latin America, inflation is always and everywhere a fiscal phenomenon.
Thomas J. Sargent (2018)

Understanding the relationship between monetary and fiscal policies is key for policy-makers. There are several approaches to doing so. A plausible way of exploring such a relationship is to consider the inter-temporal aggregate government budget constraint.[1] In essence, it implies that the nominal government debt, standardized by the price level, is equal to the expected tax revenues minus government expenditures plus seigniorage. In general, this equation has a Ricardian and a non-Ricardian interpretation.

Under Ricardian equivalence, this equation is a constraint on fiscal policy. Thus, for instance, a tax increase must eventually follow a rise in government expenditures. If agents perceive that the expected present value of net tax revenues is insufficient to back the government's nominal debt, then the government will increase its seigniorage, having no other source of income. Such an increase would put pressure on the price level, in a situation known as fiscal dominance (Sargent and Wallace, 1981).

A strand of the literature entertains the possibility of non-Ricardian policies. Under this framework, price levels adjust to satisfy a particular equilibrium condition, the intertemporal budget constraint, after the government sets its desired deficit or surplus level. Consequently, for instance, incremental changes in government expenditures will not necessarily lead to tax adjustments. Instead, such increments can directly lead to raises in price levels. This interpretation is central to the fiscal theory of the price level (e.g., Leeper, 1991).[2]

Theories of inflation, however, have been almost exclusively built upon the money-demand function of Cagan (1956). Recent empirical work (see Benati, Lucas, Nicolini, and Weber, 2019) has found that, in many countries, other specifications of the money-demand function do a better job of characterizing the long-run relation between money demand and inflation

[1] It is aggregate in the sense that it includes the fiscal and central bank's budget constraints.

[2] This theory has been subject to important criticisms (e.g., Buiter, 1987).

expectations than Cagan's money-demand function. Against this backdrop, our main goal is to explore whether the choice of money-demand function is important for obtaining inflation models that yield better predictions and policy recommendations. To this end, we propose and estimate an extension of the model in Sargent, Williams, and Zha (2009), abbreviated as SWZ henceforth, on Mexican monthly inflation data from February 1969 to July 2019 and contrast the model's predictions for the relationship between three key macroeconomic variables in Mexico: inflation, inflation expectations (i.e., beliefs), and seigniorage-financed deficits.

In the model, the deficit follows a probability distribution with a mean and a variance that, in turn, follow Markov-switching regimes. This allows us to obtain model-implied inflation expectations, seigniorage-financed deficits, and the regime dynamics determining such deficits. In particular, we find that the choice of functional form for the money demand has a substantial effect on the model's dynamics and equilibria, and on the resulting relationship between fiscal deficits and seigniorage.

We have a keen interest in exploring the relationship between inflation dynamics and the deficit regimes predicted by the model, because this interaction leads to predictions regarding the stability of inflation expectations. As we will see, if inflation beliefs rise beyond some threshold, expectations may become explosive but not necessarily unstable. This is because our model features stable equilibria at very high levels of inflation, and convergence to these equilibrium levels can be considered as explosive behavior if such equilibria involve hyperinflation. Thus, the money-demand function matters for comprehending economic behavior at high levels of inflation. On the other hand, the money-demand function contains predictions on the relationship between inflation and seigniorage. Cagan's paradox weakens the relationship between both and thus may obfuscate the real relationship between historical deficits and inflation. In fact, because our model is free of Cagan's paradox, we can obtain a better match between historical deficits and model-implied seigniorage-financed deficits in Mexico. Departing from Cagan's money-demand function, however, has costs in terms of more difficult estimation and computation of equilibria. In our case, the former involves numerical solutions of equations at each step of the maximum likelihood estimation and the latter, a computationally intensive Monte Carlo integration to obtain the function whose solution determines the equilibrium.

Summarizing some of our main results on Mexican data, we have the following remarks. First, the 1982, 1987, and 1994 crises were closely associated with regime-switching to higher mean deficit levels. The Mexican government switched to lower deficit-mean regimes only after such crises. We interpret regime-switching toward lower deficit-mean states as the product of fundamental reforms, which had varying degrees of success.[3]

Second, inflation expectations increased continually during the 1980s, adjusting toward high or very high and stable expectations equilibria. However, they regained their stability in the aftermath of the 1994 crisis, a.k.a. the Tequila Crisis. After the last switch to the lowest deficit-mean regime, the probability of being in such a regime has generally stayed the highest among all probabilities of the deficit-mean states. This is a result of successful reforms implemented during the late 1990s, which brought about stability of inflation and its expectations in Mexico.

Third, we explore the extent to which the model-implied variables were related to key economic events. In several episodes, fiscal adjustments proved insufficient to reduce and stabilize inflation and its expectations. It can be argued that further measures, such as income policies and external debt renegotiation, were also important for attaining their stability. Our model predicts that the size of the fiscal retrenchment matters, sometimes requiring seigniorage-financed deficits to be reduced to levels below those at the beginning of the inflationary episode, just to restore inflation to its previous levels. While the level of inflation expectations is important to achieve these conclusions because it implies a lower or higher demand for real balances, it is also important to determine whether they are in the domain of attraction of a high or low equilibrium level of inflation expectations.

We proceed as follows: we begin with a brief review of the relevant literature, and we describe the model, while setting the equilibrium definitions. Then, we estimate the model with Mexican data, and we finish by comparing the model predictions with the the Mexican history of inflation and stabilization reforms.

2 Literature Review

We divide our literature review into two parts. In the first, we discuss general papers on the subject. In the second part, we review some papers focusing

[3]More generally, in most economic crises, regardless of where the difficulties originate, public finances eventually absorb a large part of, if not all, the costs, leading to significant increases in deficits and public debt levels.

on the Mexican economy.

Cagan (1956), in his classic paper, studies dual inflation equilibria; maximizes seigniorage given a semi logarithmic money-demand function and a mechanism for forming inflation expectations.[4] He shows that for any feasible level of seigniorage, there are two inflation equilibria resulting from the shape of the demand for money. One equilibrium is associated with low inflation, and the other is associated with high inflation.[5] This result is akin to the celebrated Laffer curve, in which, for the same level of tax revenue (seigniorage), there are two equilibria involving a low- and a high-inflation tax rate. This would imply that monetary authorities expand the money supply beyond the optimal level of seigniorage during episodes of very high inflation. But, as Benati (2018) underscores, the empirical evidence for an inflationary Laffer curve is scant.

Wallace and Sargent's 1981 seminal article provides a framework with which to assess several features of the relationship between monetary and fiscal policies. They examine the implications of the inter-temporal aggregate government budget constraint.[6] In particular, they characterize a situation in which agents deem that the expected net tax revenues will be insufficient to back the nominal government debt. In such a situation, the government might appeal to seigniorage to back its debt, thereby using monetary policy to fulfill its fiscal needs. This framework, although quite simple, has led to powerful insights like fiscal dominance.

Bruno and Fischer (1990) study the stability of dual inflation equilibria under seigniorage-financed deficits. The authors highlight that, in general, under adaptive (rational) inflation expectations, the high-inflation equilibrium is unstable (stable), and the low-inflation equilibrium is stable (unstable).[7]

[4] Cagan (1956) does not consider a budget constraint.

[5] The exception is the point at which seigniorage is at its maximum, which is associated with a unique inflation equilibrium.

[6] Again, it is aggregate in the sense that it includes the fiscal and central bank's budget constraints.

[7] The stability of the two inflation equilibria depends on the product of the semi-elasticity of money (λ) demand times the adaptive expectations parameter (ν). Specifically, if their product is less than one ($\lambda\nu < 1$), then the low-level inflation equilibrium will be stable, and the high-level inflation equilibrium will be unstable. On the other hand, if their product is greater than one ($\lambda\nu > 1$), then the low-level inflation equilibrium will be unstable, and the high-level inflation equilibrium will be stable. Intuitively, if $\lambda\nu < 1$, then the agent will respond and/or adapt slowly to changes in inflation expectations. Moreover,

Akin to Cagan (1956), they use a semi logarithmic money-demand function and work with adaptive expectations but include a government budget constraint. In their model, the economy might find itself trapped in a high-inflation equilibrium, although a low-inflation equilibrium is feasible with the same public financing needs. There is a unique equilibrium under some conditions, such as when allowing for bond financing of deficits, while fixing the nominal growth rate of money. Intuitively, the government gains flexibility by allowing for bond financing, and it gains some credibility by fixing the nominal growth rate of money. This will work if bonds only finance temporal deviations in government net expenses. They underscore that dual equilibria are avoidable because they are the product of the operating rules set in place.

Bruno (1989), using Bruno and Fischer's 1990 model, is concerned with designing economic reforms.[8] He conceives of a reform as a planned transition from a high-inflation equilibrium to a low-inflation one, which is Pareto superior. He applies this rationale to an inflation-stabilization program implemented in Israel. Such a program entailed corrections in the government budget and external accounts, and the implementation of wage and price controls. Beyond the fiscal adjustment necessary to bring inflation to a low and stable level, further actions played a role in coordinating inflation expectations toward a low-inflation equilibrium.

SWZ explore the relationship among inflation, inflation expectations, and seigniorage-financed deficits in five South American economies: Argentina, Bolivia, Brazil, Chile, and Peru. Their framework consists of a demand for money, an inter-temporal public budget constraint, and a formation mechanism for inflation expectations.[9] In addition, they consider a regime-switching process that affects the distribution of the seigniorage-financed deficit. In particular, they examine the stability of inflation expectations formed under certain learning mechanisms, while also considering an inter-temporal budget constraint, a money-demand function, and a Markov chain

if inflation has low levels, then its response will be sufficiently fast to maintain stability. However, if inflation has high levels, then its response will not be sufficiently fast, and the equilibrium point will be unstable. On the other hand, if $\lambda\nu > 1$, then the agent will respond and/or adapt quickly to changes in inflation expectations. Moreover, if inflation is low, then the agent's response will not be sufficiently fast to maintain stability. However, if inflation is high, then the agent's reaction will be sufficiently fast, and the equilibrium will be stable.

[8] Although Bruno and Fischer (1990) published their paper after Bruno (1989) did, a working paper was available a few years prior.

[9] One can think of their demand as an approximation of a semi logarithmic demand.

that induces the economy's mean deficit to switch between regimes. More specifically, they explore the extent to which changes in inflation and inflation expectations are related to switching between deficit regimes.

They document that regime switching typically comes about when economies have either fallen or confront a high probability of falling into the domain of attraction of unstable inflation expectations. These authors distinguish between two types of reforms. First, reforms that take inflation expectations to a stable level but without regime switching taking place, are denominated as cosmetic reforms. Second, reforms that lead to stable inflation expectations due to regime switching in the deficit-mean, are called fundamental reforms.

In Benati, Lucas, Nicolini, and Weber (2019), long-term estimations of money demand are performed for several countries. They find that neither the popular Cagan semi log nor the log-log specifications are appropriate for a large number of countries, including Mexico. They propose either a log-log functional form with borrowing constraints, or an approximation based on the Selden-Latané money-demand function.

Benati (2018) shows that replacing Cagan's semi log for a log-log specification in a standard inflation model yields different predictions regarding equilibria and dynamics, and that such a model can display explosive inflationary behavior even when steady-state equilibria are well defined. In this paper, we analyze the inflationary dynamics of a SWZ model with a variation of the Selden-Latané money-demand function and a more realistic deficit-volatility function.

Among the literature related to Mexico, we highlight the following papers. Ortiz (1991) discusses the stabilization program following the 1982 crisis. He argues that the program brought inflation down and avoided a recession. It distinctly involved structural reforms alongside fiscal and income policies, namely trade liberalization, deregulation, and the privatization of some government-owned firms. Two elements were central to the program: income policies and external debt negotiation.

Gil-Díaz and Carstens (1996b) explore how the fiscal and trade reforms in Mexico could have led its economy into the crisis associated with the December 1994 exchange-rate devaluation. They contend that researchers often mention the political events in 1994 in passing or as a trigger but not as a source. After examining some of the hypotheses advanced, the authors conclude that the crisis had political origins. They argue that certain factors contributed to the crisis, including the fixed nominal exchange-rate regime and an upsurge in international transactions (see also Gil-Díaz and Carstens, 1996a).[10]

Ramos-Francia and Torres (1994) assess the role of monetary policy in the Mexican disinflationary process from 1994 to 2003. They argue that once an economy establishes a sustainable fiscal position, an inflation-targeting framework functions as a disciplinary mechanism for monetary policy. They describe key measures taken to stabilize the economy after the 1994 crisis and argue that these measures prevented fiscal dominance. They contend that the central bank's policy responses were consistent with inflation-targeting principles.

Meza (2018) analyzes the monetary and fiscal history of Mexico using the model described in Sargent and Wallace (1981). He studies the 1960–2007 period, and assesses the model's ability to explain the 1982 and 1994 crises. He claims that it succeeds at explaining the 1982 crisis but fails to do so for the 1994 crisis. In addition, he argues that the constitutional changes—concerning the relationship between the government and the central bank—and the policy choices made in the aftermath of the 1994 crisis were in line with a transition from fiscal dominance to an operationally independent central bank.

López-Martín, Ramírez de Aguilar and Sámano (2020) estimate the SWZ model for the Mexican economy and added variables such as the exchange-rate and Mexican bond spreads in the formation mechanism for inflation expectations.

3 The Model

We start by specifying a money-demand function:

$$\frac{M_t}{P_t} = \frac{1}{\gamma}\lambda\left(\frac{P^e_{t+1}}{P_t}\right), \tag{1}$$

where the nominal money demanded is denoted as M_t which is a percentage of the output at period t; P_t is the price level; and P^e_{t+1} is the expected price level in the next period. The function $\lambda(\cdot)$ captures the sensitivity of real

[10] Musacchio (2012) argues that the excessive eagerness of foreign investors and weak regulation of the banking system led to a buildup of vulnerabilities that left Mexico exposed to variations in investors' sentiment. The political events in Mexico along with changes in U.S. monetary policy led to significant changes in investors' perception of Mexico's future.

money demand to changes in expected inflation, and γ is a scale parameter, with $\gamma > 0$. The function $\gamma(\cdot)$ has the following form:

$$\lambda\left(\frac{P_{t+1}^e}{P_t}\right) = \frac{\lambda_0}{1 + \lambda_1(P_{t+1}^e/P_t - 1)},$$

where $0 < \lambda_0 < 1$ and $\lambda_1 > 1$. It is a form of Selden's 1956 and Latané's 1960 money-demand function. This can be verified by letting $a = \gamma/\lambda_0$ and $b = \gamma\lambda_1/\lambda_0$ and then rewriting equation (1) as $M_t/P_t = 1/(a+b(P_{t+1}^e/P_t - 1))$. Under this parameterization, λ_0 determines the demand for real balances when expected inflation is zero, while λ_1 determines jointly the money-demand elasticity and the lower bound for expected inflation.

This functional form behaves similarly to the log-log money-demand function of Meltzer (1963) when the expected inflation is high, but it does not explode when the latter is low and close to zero, i.e., when P_{t+1}^e/P_t approaches one. Instead, it increases in a non-explosive way toward λ_0, as expected inflation reaches zero. Moreover, this money-demand function, as advocated by Benati, Lucas, Nicolini, and Weber (2019), is consistent with long-run estimations of money demand for several countries, including Mexico. In effect, many of these countries have experienced episodes of very high inflation, or even hyperinflation, but also episodes of low inflation, with inflation rates close to zero.

A log-log money-demand function counter-factually predicts overly high levels of real money demand when inflation expectations fall very close to zero. Intuitively, money demand cannot increase to infinity unless individuals have access to unlimited credit. Our money-demand function predicts explosive behavior for inflationary expectations below zero, that is, when expected inflation approaches the value $P_{t+1}^e/P_t \to 1 - 1/\lambda_1$. Should the need arise, this could be fixed by introducing borrowing constraints, but for our data and estimated parameters, both the realized inflation and expected inflation are well above this critical level. Benati, Lucas, Nicolini, and Weber (2019) propose setting real money demand to a constant, in order to introduce borrowing constraints for log-log specifications to rule out explosive behavior for very low levels of expected inflation. Our money-demand function does not allow explosive behavior for any positive values of expected inflation (i.e., $\beta_t > 1$) and can be thought of as an approximation of a log-log money-demand function with borrowing constraints.

In the SWZ model, the money-demand function is a linear approximation of Cagan's semi logarithmic money-demand function. Recall that

Cagan's money-demand function is $M_t/P_t = \exp(\lambda \mathbb{E}_t \pi_{t+1})$, where $\mathbb{E}_t \pi_{t+1}$ is the conditional expected inflation. SWZ employ an approximation of this demand based on the Taylor expansion of the exponential function: $M_t/P_t \approx 1 - \lambda \mathbb{E}_t \pi_{t+1}$. In this money-demand function, λ is a scalar interpreted as the semi-elasticity of the money demand with respect to the expected inflation. Due to the linearity of this functional form, the demand for real balances can take negative values for very high levels of expected inflation. Negative money-demand levels are handled in SWZ by resetting inflation and possibly expectations to a low level, as a type of cosmetic reform. Our money-demand function cannot take negative values, so cosmetic reforms of this type are unnecessary. In Mexico's case, we do not lose generality by doing so because we did not find cosmetic reforms of this type in our estimations of the original SWZ model with Mexican data. We summarize the behavior of these money-demand specifications in figure 1.

Our money-demand function, of course, implies that the demand for real balances M_t/P_t depends negatively on the expected price level. In effect, higher inflation implies a higher opportunity cost of holding money. The money supply is given by the government's budget constraint:

$$M_t = \theta M_{t-1} + d_t(m_t, \varsigma_t, d_{t-1}) P_t. \tag{2}$$

The parameter θ adjusts the money supply for the growth in real output and for direct taxes on cash balances, if any; its value satisfies $0 < \theta < 1$. A lower bound on θ is given by $1 - 1/\lambda_1 < \theta$, and it is sometimes a condition for the existence of an equilibrium, as described below. The government spends the seigniorage obtained by creating money on deficit financing d_t. The deficit financed with seigniorage has the following dynamics:

$$d_t(m_t, \varsigma_t, d_{t-1}) = \bar{d}(m_t) + \varepsilon_d(\varsigma_t, d_{t-1}),$$

in which we assume that the deficit has an average level $\bar{d}$ which depends on a regime m_t. Deficit shocks are captured by $\varepsilon_d(\varsigma_t, d_{t-1})$, similarly depending on a volatility regime and on the deficit during period $t-1$. This shock induces a log-normal conditional distribution for the deficit with a mean equal to $\log \bar{d}(m_t)$ and variance $\sigma_d^2(\varsigma_t, d_{t-1}) = \sigma_d^2(\varsigma_t) d_{t-1}^{\vartheta}$ whenever the deficit is positive. Our volatility function takes this form because time-varying deficit volatility is a noteworthy feature of fiscal deficits that captures the distortions and the weakened fiscal sustainability caused by high deficits. In particular, empirical works such as Cevik and Teksoz (2014) and Agnello and

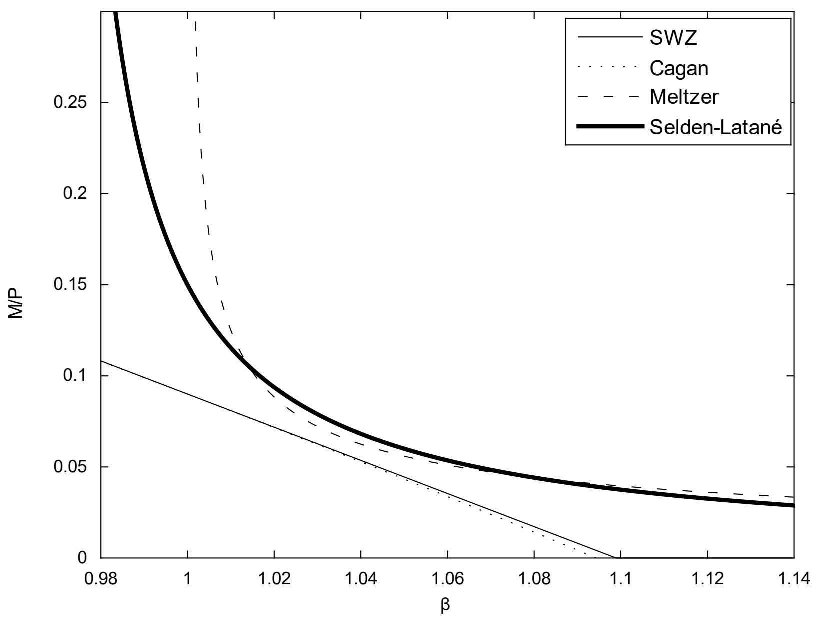

Figure 1: **Money-Demand Functions**. Note: The linear money-demand function in SWZ is shown here with $\lambda = 0.91$, which is close to Cagan's semi log money-demand function, displayed here with the same parameter value. Meltzer's log-log money-demand function is shown with an elasticity of 0.5, and re-scaled to show that, for high levels of inflationary expectations, it can be very close to our version of Selden-Latané's money-demand function, here shown with $\lambda_0 = 0.15$ and $\lambda_1 = 30$. Source: Own calculations.

Sousa (2013) have documented that the deficit's volatility function depends on its size. Time-varying deficit volatility is a crucial stylized fact because when it is high, it diminishes the real effects of fiscal policies and increases their inflationary consequences, among other distortions. Deficit volatility increases a country's borrowing costs, which in turn increases present and future deficits, worsening the deficit's inflationary impact. In addition, we found that our volatility function helps to better identify regimes and to decrease the number of volatility regimes.

The deficit volatility also depends on the parameter σ_d^2, which, in turn, is a function of a state that changes according to the regime ς_t. In our results, the volatility state spends most of the time in the lower level and only briefly increases to the high level. The high-volatility regime may be capturing external shocks to the economy, by temporarily allowing deficit shocks to take higher values than usual. Defining the joint deficit state as $s_t \equiv (m_t, \varsigma_t)$, we can write the deficit as depending on just two arguments: $d_t(s_t, d_{t-1}) = d_t(m_t, \varsigma_t, d_{t-1})$.

The deficit distribution has a density function denoted by $p_d(\varepsilon_d|s_t, d_{t-1})$ and it takes the form

$$p_d(\varepsilon_d|s_t, d_{t-1}) = \frac{\exp\left(\frac{-\left[\log(\bar{d}(m_t)+\varepsilon_d)-\log(\bar{d}(m_t))\right]^2}{2\sigma_d^2(\varsigma_t, d_{t-1})}\right)}{\sqrt{2\pi}\sigma_d(\varsigma_t, d_{t-1})(\bar{d}(m_t)+\varepsilon_d)} \text{ if } \bar{d}(m_t)+\varepsilon_d > 0, \tag{3}$$

and 0 in other cases. We also assume that the elements of the joint deficit state (m_t, ς_t) follow independent Markov chains, with the following transition probabilities:

$$\begin{aligned} Pr(m_{t+1} = j|m_t = i) &= p_{i,j} \text{ where } i, j = 1, ..., m_h & (4) \\ Pr(\varsigma_{t+1} = j|\varsigma_t = i) &= q_{i,j} \text{ where } i, j = 1, ..., \varsigma_h. & (5) \end{aligned}$$

There are a total of $m_h \times \varsigma_h$ possible states. We stack the probabilities defined in (3) and (4), yielding matrices denoted by Q_m and Q_ς, respectively, where $[Q_m]_{i,j} = p_{i,j}$ and $[Q_\varsigma]_{i,j} = q_{i,j}$. Additionally, we denote the transition probability matrix of the joint state $s_t \equiv (m_t, \varsigma_t)$ as Q_s. Because we have assumed that the Markov chains are independent, we get $Q_s = Q_m \otimes Q_\varsigma$, where $\otimes$ denotes the Kronecker product between the matrices.

The model is completed by specifying the formation mechanism of inflation expectations (i.e., beliefs). Typically, under rational expectations, the expected level of prices P_{t+1}^e is set equal to its mathematical expectation $\mathbb{E}_t\left[P_{t+1}\right]$. Instead, this model assumes a mechanism with constant-gain learning. Defining

$$\pi_{t+1}^e \equiv \frac{P_{t+1}^e}{P_t} = \beta_t \tag{6}$$

we then have the following adaptive expectations mechanism:

$$\beta_t = \beta_{t-1} + \nu\left(\pi_{t-1} - \beta_{t-1}\right), \tag{7}$$

where $0 < \nu \ll 1$ and π_t[11] denotes gross inflation[12], defined as P_{t+1}/P_t.[13] This learning mechanism is relevant for the stability of inflation expectations and facilitates the construction of the likelihood function. We use the terms "inflation expectations", "inflation beliefs", and "beliefs" interchangeably.

Additional Restrictions on Inflation Expectations

Consider the demand for money, equation (1), the government budget constraint, represented by equation (2); and the formation of inflationary expectations, denoted by equation (7). One can then obtain the following expression for the equilibrium inflation: [14]

$$\pi_t = \frac{\theta\lambda\left(\beta_{t-1}\right)}{\lambda\left(\beta_t\right) - \lambda d_t\left(s_t, d_{t-1}\right)}, \tag{8}$$

for all t, provided that the numerator and denominator are positive. In this context, β_t and β_{t-1} must satisfy the following inequalities:

$$\begin{aligned} \beta_t &> 1 - \frac{1}{\lambda_1} && (9) \\ \lambda(\beta_t) - \gamma d_t(s_t, d_{t-1}) &> \delta\theta\lambda(\beta_{t-1}), && (10) \end{aligned}$$

[11]The expression $x \ll y$ means that x is much smaller than y.

[12]E.g., a gross inflation of 3% is denoted by 1.03, instead of the more common 0.03.

[13]Constant-gain learning means that ν is constant. There are other, more general rules. For example, there are some for which such a parameter might be a function of the probability of the state. One then increases (decreases) ν if the state has a high (low) probability of taking place. Intuitively, one pays more (less) attention to more (less) probable states.

[14]Substitute the inflation belief β_t for the demand for money $M_t/P_t = \lambda(\beta_t)/\gamma$, and then rewrite the money-demand function as $M_t/P_t = \theta(M_{t-1}/P_{t-1})/\pi_t + d_t$. Plug the first equation into the second for t and $t+1$ to obtain $\lambda(\beta_t)/\gamma = \theta(\theta(\beta_{t-1})/\gamma)/\pi_t + d_t$.

almost surely for all $t > 0$. Restriction (9) sets a lower bound for inflation expectations and, together with equation (1), implies that the real money stocks are positive and finite for all $t > 0$. Restriction (10) sets δ^{-1} as an upper bound for gross inflation. This bound is a necessary condition for the existence of a self-confirming equilibrium (SCE), which we will define later. It could be enforced through a cosmetic reform, i.e., an inflation shock with variance σ_π, applied instantaneously whenever inflation surpasses the bound, to ensure that inequality (10) holds and preventing $\pi_t \to \infty$.

Restriction (10) ensures that gross inflation is always positive, but inflation can still be negative. In equation (8), if d_t is small enough and $\lambda(\beta_{t-1})/(\lambda(\beta_t) - \gamma d_t)$ is close or equal to one then the equilibrium inflation will be around θ, which is less than one. As equation (10) ensures that the denominator of equation (8) is always positive, expectations may not satisfy equations (9) and (10) at the same time unless $1 - 1/\lambda_1 < \theta$. Our model allows equilibrium inflation and expectations to be slightly negative, as long as equation (9) continues to hold.

4 The Deterministic Model and Its Steady-State

In this section, we consider a deterministic version of the model. By this, we mean that the deficit-mean state m is fixed (i.e., known), and shocks ε_d are set equal to zero for all t. This implies that the volatility and the state ς_t are inconsequential in computing the deterministic steady-state and, thus, we have $d_t = \bar{d}(m)$ for all t. Likewise, inflation expectations are already settled at this steady-state, so even for adaptive expectations $\beta_t = \pi^e_{t+1} = \pi_{t+1}$ for all t. Next, consider the money-demand function, equation (1), and the government budget constraint, equation 2, which yield, under these conditions:

$$\frac{M_t}{P_t} = \frac{1}{\gamma}\lambda(\pi_{t+1}), \tag{11}$$

$$\frac{M_t}{P_t} = \theta\frac{M_{t-1}}{P_{t-1}}\frac{1}{\pi_t} + \bar{d}(m). \tag{12}$$

The above imply the following equation:

$$\pi_t = \frac{\theta\lambda(\pi_t)}{\lambda(\pi_{t+1}) - \gamma\bar{d}(m)}. \tag{13}$$

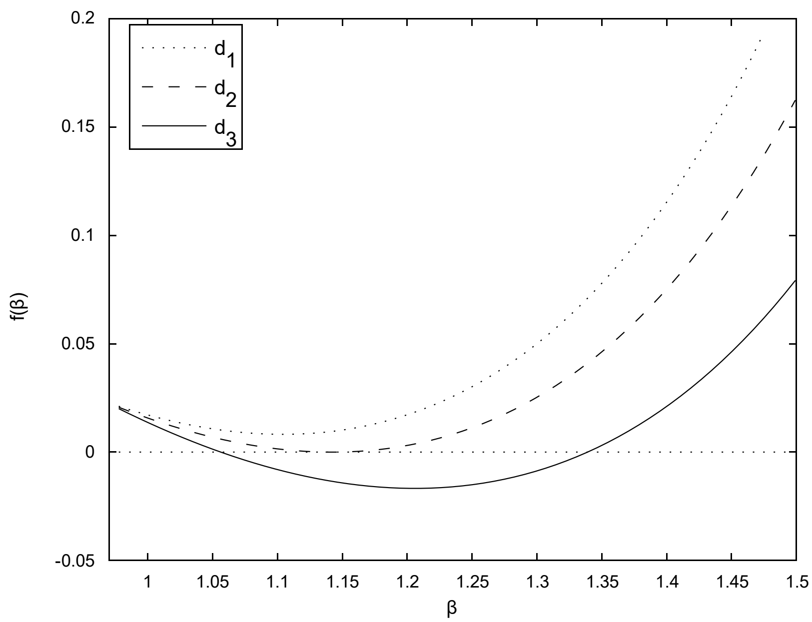

Figure 2: **Deficits and Deterministic Stationary State Equilibria**. Note: For regimes with a low deficit-mean such as d_3, the model has two SSEs, defined as the zeros of the function in equation (14). When the mean deficit increases to d_2, the number of SSEs is reduced to one, while for values of the mean deficit greater than d_2, such as d_1, the model will have no SSEs. Source: Own calculations.

If $\pi_{t+1} = \pi_t = \pi(m)$, then we obtain the following nonlinear equation with inflation as the unknown variable:

$$\pi = \frac{\theta\lambda(\pi)}{\lambda(\pi) - \gamma\bar{d}(m)}. \tag{14}$$

This equation might have zero, one, or two solutions. We define a stationary state equilibrium (SSE), denoted as π^*, as a zero of equation (14)

This equation does not have a closed-form solution in our setup, but its zeros can be readily calculated numerically. By equation (9), first, we know that a solution must satisfy $\pi^* > 1 - 1/\lambda_1$, and by equation (10), we can also determine that a solution exists if

$$\gamma \bar{d}(m) < \lambda(\pi). \tag{15}$$

These are necessary but not sufficient conditions. A tighter upper bound for the deficit can be found numerically: there is a maximum deficit level d_{max}, such that a deterministic steady-state exists. This will imply a maximum level of (low) steady-state inflation, denoted by π^*_{max}. For deficit levels lower than d_{max}, two solutions will exist, which we denote $\pi^*_1(m) < \pi^*_2(m)$. The first solution is the low-inflation SSE, and the second is the high-inflation SSE. On the other hand, when $\bar{d}(m) = d_{max}$, there will be only one SSE, denoted as $\pi^*_1(m) = \pi^*_2(m) = \pi^*_{max}$. When the mean deficit approaches zero, the solution of equation (14) becomes unique. In particular, when the mean deficit is zero, then the unique solution is $\pi^* = \theta$, which can only be an equilibrium if equation (9) is satisfied–that is, if $1 - 1/\lambda_1 < \theta$.

When the deficit-mean is close to but not zero, the second (high) solution $\pi^*_2(m)$ to equation (14) can be greater than $1/\delta$, exceeding inflation's upper bound given by equation (10); thus, this solution is not an SSE, leaving $\pi^*_1(m)$ as the unique equilibrium. In figure 2, we show that for a low deficit-mean level such as d_3, the model has two SSEs. As the deficit-mean increases to $d_2 = d_{max}$, the two SSEs become close to each other and eventually become the same. For any deficit-mean level greater than d_2, such as d_1, the model does not have an SSE.

For our estimated parameters with Mexican data, SSEs do not exist for the highest deficit-mean state, while the remaining states with lower deficit-mean have two SSEs. As SSEs are used in the estimations as approximations of self-confirming equilibria (SCEs, to be defined below), we use the value π^*_{max} in our computations whenever SSEs do not exist.

5 Dynamics around Self-Confirming Equilibria

More general equilibria can be calculated, where the stochastic shocks on the deficit and the Markov chains affecting the deficit distribution are not completely muted. For example, we can hold the state m as fixed and allow the shocks ε_d to impact the deficit. We provide the following definition of equilibrium, similar to the fixed-m SCE of SWZ:

Definition 1. Fixed-m self-confirming equilibrium (SCE). For each m-state, a fixed-m SCE is a probability distribution over inflation histories $\pi^T \equiv \{\pi_1, \pi_2, ..., \pi_T\}$, and $\beta(m)$, possibly non-unique, such that

$$\mathbb{E}\left[\pi_t | m_t = m \ \forall t\right] - \beta(m) = 0. \tag{16}$$

Although the mean deficit state m can change and agents have adaptive expectations, when the deficit regime process is highly persistent, a SCE represents a good approximation to the steady-state expectations.[15] Following SWZ, when agents are confident about their previous beliefs, specifically meaning that they more closely rely on them to form their beliefs for inflation in the following period (i.e., when ν converges to 0), and when the deficit regime becomes more persistent, inflation beliefs converge to the solution of an ordinary differential equation of the following form:[16]

$$\dot{\beta} = \hat{G}(\beta, m), \tag{17}$$

where a fixed-m SCE is a fixed point of the function $\hat{G}(\beta, m)$ for each m. Thus, inflation beliefs converge to a limit that only depends on m.

We now compute this SCE. Let us start by writing down the evolution of inflation. First, since inflation is bounded by δ^{-1} (see equation (10)), we need to specify what happens when inflation reaches or exceeds such a bound. Let us rewrite the inflation bound as one on the deficit $d_t(s_t, d_{t-1}) < \omega(\beta_t, \beta_{t-1})$, where

$$\omega(\beta_t, \beta_{t-1}) \equiv \frac{\lambda(\beta_t) - \delta\theta\lambda(\beta_{t-1})}{\gamma}. \tag{18}$$

Then, when inflation is below its bound, its equilibrium level will be determined by the equilibrium condition (8); otherwise, a cosmetic reform kicks in, and inflation will be reset to the lowest equilibrium $\bar{\pi}_1^*(s_t)$ given the state m as follows:

[15] Intuitively, if the process is highly persistent, m_t will remain in the same regime state for a long time. This will allow the adaptive expectations mechanism to get closer to π_t, resulting in $\mathbb{E}[\pi_t | m_t = m \ \forall t] - \beta(m) = 0$ as a good enough approximation.

[16] As shown by SWZ, the convergence is weak, i.e., convergence in distribution.

$$\begin{aligned}\pi_t =& \iota\big(d_t(s_t, d_{t-1}) < \omega(\beta_t, \beta_{t-1})\big)\frac{\theta\lambda(\beta_{t-1})}{\lambda(\beta_t) - \gamma d_t(s_t, d_{t-1})} \\ &+ \iota\big(d_t(s_t, d_{t-1}) \geq \omega(\beta_t, \beta_{t-1})\big)\bar{\pi}_1^*(s_t).\end{aligned} \tag{19}$$

Properly speaking, $\bar{\pi}_1^*(s_t)$ is here the low conditional SCEs, but, following SWZ, we instead use the low deterministic SSE to compute equation (19).

Defining (19) with the SCEs would require solving a computationally intractable double fixed-point problem to estimate the SCEs. Ex-post, we found the low SCEs to be very close to the low deterministic SSEs when both exist. In those cases where there is no deterministic equilibrium, $\bar{\pi}_1^*(s_t)$ is replaced by π^*_{max}.

Using equation (19), we define the inflation belief error as follows:

$$\begin{aligned}g\big(\beta_t^*, \beta_t, \beta_{t-1}, &d_t(s_t, d_{t-1})\big) = \pi_t - \beta_t \\ &= \iota\left(d_t(s_t, d_{t-1}) < \omega(\beta_t, \beta_{t-1})\right)\frac{\theta\lambda(\beta_{t-1})}{\lambda(\beta_t) - \gamma d_t(s_t, d_{t-1})} \\ &\quad + \iota\left(d_t(s_t, d_{t-1}) \geq \omega(\beta_t, \beta_{t-1})\right)\bar{\pi}_1^*(s_t) - \beta_t,\end{aligned} \tag{20}$$

where ι is an indicator function. To compute the SCE, we need to define

$$\tilde{\omega}(\beta) = \omega(\beta, \beta) = (1 - \delta\theta)\lambda(\beta)/\gamma, \tag{21}$$

and note that as $\beta \to \infty$, then $\tilde{\omega}(\beta) \to 0$. We will use this result later. We can now rewrite the adaptive expectations mechanism as

$$\beta_{t+1} - \beta_t = \nu g(\pi_t^*, \beta_t, \beta_{t-1}, d_t(s_t, d_{t-1})), \tag{22}$$

or, more generally,

$$\beta_{t+\Delta} - \beta_t = \nu g(\pi_t^*, \beta_t, \beta_{t-\Delta}, d_t(s_t, d_{t-\Delta})), \tag{23}$$

which takes the form of equation (17) as $\nu \to 0$ and $\Delta \to 0$ jointly, and after taking expectations, conditioning on the state m. Then, to find the equilibrium value of β, we must evaluate the expectation

$$\mathbb{E}g(\pi_t^*, \beta_t, \beta_{t-1}, d_t(s_t, d_{t-1})) = 0, \tag{24}$$

conditioning on $m_t = m$, and then, solve for β. To help integrate out the deficit shocks and the lagged deficit, we define

$$\Psi_s(\beta, b) = \int_0^\infty \int_0^{b-\bar{d}(m)} \frac{1}{\lambda(\beta) - \gamma(\bar{d}(m) + \varepsilon_d)} dF_d(\varepsilon_d|s, d') dF_d^*(d'|s), \quad (25)$$

which will help compute the expectation of equilibrium inflation (8) given $d_t(s_t, d_{t-1}) < \omega(\beta_t, \beta_{t-1})$. The upper bound of the inner integral in Ψ_s is given by the conditioning deficit bound, and F_d^* denotes the distribution of the deficit, conditioning solely on the joint state $s_t = s$. Similarly, we define

$$\Phi_s(\beta, b) = \int_0^\infty \int_0^{b-\bar{d}(m)} dF_d\left(\varepsilon_d|s, d'\right) dF_d^*\left(d'|s\right), \quad (26)$$

This helps compute the expected value of post-cosmetic reform inflation, given $d_t(s_t, d_{t-1}) \geq \omega(\beta_t, \beta_{t-1})$. Φ_s is a cumulative distribution function that will be used to assess the probability of inflation reaching its upper bound. Recall that the deficit bound is rewritten as $\tilde{\omega}(\beta)$ when β is constant, and we evaluate these integrals from 0 to $\tilde{\omega}(\beta) - \bar{d}(m)$ to ensure they are finite. Additionally, the inflation's upper bound, together with the deficit bound, guarantee that $\Psi_s(\beta, b)$ is finite. As $\tilde{\omega}(\beta) \to 0$ if $\beta \to \infty$, Φ_s eventually decreases toward 0. Now, let us define

$$\tilde{g}(\pi_t^*, \beta, d_t) = g(\pi_t^*, \beta, \beta, d_t). \quad (27)$$

We now collect definitions to provide an expression for $\tilde{g}$. We denote $\bar{q}_{\varsigma,k}$ as the unconditional probability of the event $\varsigma_t = k$, which is an element of the ergodic distribution of Q_ς, and then

$$\begin{aligned}
\hat{G}(\beta, m) \equiv& \mathbb{E}[\tilde{g}(\pi_t^*, \beta, d_t(m_t, \varsigma_t, d_{t-1})))|m_t = m \ \forall t] \\
=& \sum_{k=1}^{\varsigma_h} \left[\theta\lambda(\beta)\Psi_{[m,k]}(\beta, \tilde{\omega}(\beta))\right] \bar{q}_{\varsigma,k} \\
&+ \sum_{k=1}^{\varsigma_h} \bar{\pi}_1^*(k) \left[1 - \Phi_{[m,k]}(\beta, \tilde{\omega}(\beta))\right] \bar{q}_{\varsigma,k} - \beta.
\end{aligned}$$

Note that the final expression indeed has the form of (17). The zeros of $\hat{G}(\beta(m), m) = 0$ define a fixed-m SCE. Following SWZ, as $\eta \to 0$, the expectations sequence $\{\beta_t\}$ converges weakly to a random variable that is the solution of the ordinary differential equation

$$\dot{\beta} = \hat{G}(\beta, m), \tag{28}$$

for a broad class of probability distributions of $\varepsilon_d(s_t, \bar{d}(m))$, including the ones we considered here and in the aforementioned paper. We refer the interested reader to Kushner and Yin (1997) for further details. In particular, for each m based on equation (28), there exists at least one conditional SCE.

Proposition 1. *If* $1 - 1/\lambda_1 < \theta$*, there exists at least one conditional fixed-*m *SCE for every* m.

Proof: We need to show that equation (28) has at least one root at zero for every m. First, $\hat{G}(\beta, m)$ has a bounded support, open on the lower bound at $1 - 1/\lambda_1$ by equation (9), and open at the upper bound at $1/\delta$ by equation (10). By the properties of Ψ_s and Φ_s, $\hat{G}$ is bounded and continuous inside its support. Thus, through the intermediate value theorem, we only need to show that $\hat{G}(\beta, m)$ has at least one sign change. At the upper bound, as $\beta \to 1/\delta$ and as $\delta \to 0$, or equivalently as $\beta \to \infty$, then $\Psi_s, \Phi_s \to 0$ and $\hat{G}(\beta, m) \to -\infty$. However, at the lower bound, as $\beta \to 1 - 1/\lambda_1$, then $\tilde{\omega}(\beta) \to \infty$ and $\hat{G}(\beta, m)$ has a positive and finite limit: Expected gross inflation is finite because inflation is bounded. This expectation is always greater than β's lower bound if $1 - 1/\lambda_1 < \theta$, as θ is the lowest value an SSE can take. Note that the second term is determined by the SSE. If the bound on θ holds, the sum of the first two terms will be greater than the bound even if the first term is at the bound. On the other hand, the third term, $-\beta$, approaches its upper bound, which is smaller than expected inflation in absolute value. Intuitively, as the lowest values of β are not equilibria, they have to increase; therefore, $\hat{G}(\beta, m)$ is positive at β's lower bound. As $\hat{G}(\beta, m)$ always becomes negative at the upper bound, it is positive at the lower bound, and it is continuous, we can conclude that it crosses zero at least once by the intermediate value theorem. *Q.E.D.*

We find that the model can have up to three SCEs. We denote these equilibria as follows: first, a low-inflation stable SCE denoted by $\beta_1^*(m)$, which is typically very close to $\pi_1^*(m)$, the low-inflation deterministic SSE; a high-inflation unstable SCE denoted by $\beta_2^*(m)$; and a very high-inflation stable SCE denoted by $\beta_3^*(m)$. The resulting equilibria are a consequence of the interaction of a nonlinear budget constraint and a nonlinear money-demand function.

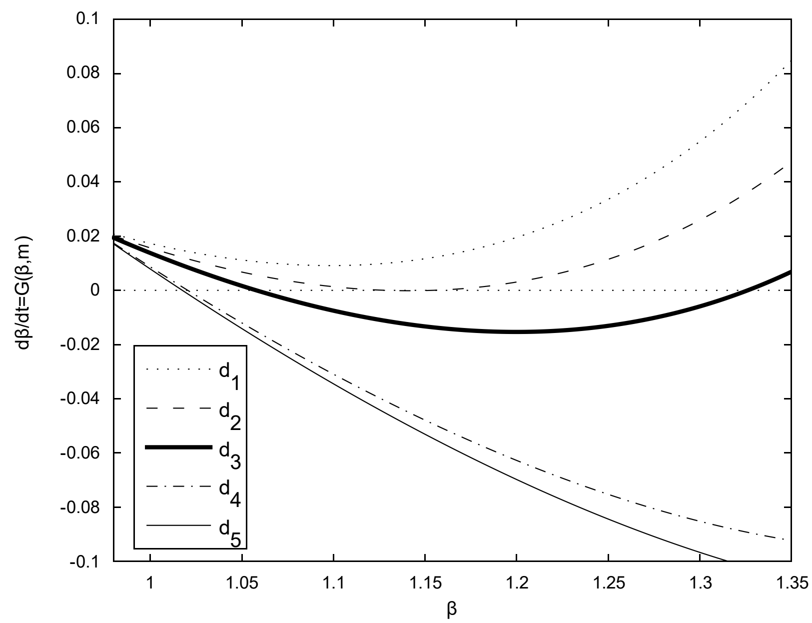

Figure 3a: **Deficits and Conditional Self-Confirming Equilibria**. Note: For intermediate levels of the mean deficit, such as d_3, the model has three conditional SCEs: Two of them are visible in this figure and the third one is visible in figure 3b. d_2 and d_4 have two equilibria, whereas d_1 and d_5 have one equilibrium each. Levels d_1 to d_4 have an equilibrium with very high inflationary expectations; they are visible in figure 3b. The lowest mean deficit level, d_5, only has one SCE, shown here at a very low level of inflationary expectations. Source: Own calculations.

We depict the typical situations that can arise depending on the state m (i.e., the level of the deficit-mean) in figures 3a and 3b. These figures have been constructed with $\lambda_0 = 0.30$, $\lambda_1 = 30$, $\vartheta = 2$, $\delta = 0.01$, $\theta = 0.99$, and $\gamma = 1$. The deficit levels d_1 to d_5 are respectively 0.0080, 0.0075, 0.0070, 0.0055, and 0.0053. We see that for the highest deficit-mean, there is only one equilibrium $\beta_3^*(m_1)$, and it is stable, i.e. $\dot{\beta} > 0$ for $\beta < \beta_3^*(m_1)$, and $\dot{\beta} < 0$ for $\beta > \beta_3^*(m_1)$. The level d_2 has two equilibria: The first one, called $\beta_1^*(m_2)$, is unstable if $\beta > \beta_1^*(m_2)$, and the second one is $\beta_3^*(m_2)$ and is stable. For the level d_3, the model has three equilibria, $\beta_1^*(m_3)$ and $\beta_3^*(m_3)$ are stable, but $\beta_2^*(m_3)$ is unstable, i.e. $\dot{\beta} < 0$ for $\beta < \beta_2^*(m_3)$ and $\dot{\beta} > 0$ for $\beta > \beta_2^*(m_3)$. The fourth level d_4 has two equilibria: A stable low equilibrium $\beta_1^*(m_4)$, and a very high equilibrium $\beta_3^*(m_4)$ which becomes unstable as soon as $\beta < \beta_3^*(m_4)$.

The fifth state has only a stable low equilibrium $\beta_1^*(m_5)$. It is also noteworthy that, when the economy is in the three-equilibria situation, (i.e. $d_4 < d < d_2$), and if the model switches to a higher mean deficit regime inside this interval, $\beta_1^*(m)$ increases, but $\beta_2^*(m)$ decreases. Accordingly, the domain of attraction of $\beta_1^*(m)$ shrinks, whereas the one belonging to $\beta_3^*(m)$ expands, increasing the probability of jumping to the domain of attraction of the latter. Such a jump is defined below as an escape event. Analytically, we have $\beta_2^*(m') > \beta_2^*(m'')$, whenever $d_4 < d(m') < d(m'') < d_2$. A higher deficit-mean regime may not only lead to a greater level of inflation but to a smaller (bigger) interval in which inflation expectations are in the domain of attraction of the low- (very high-) level SCE, and thus, a higher probability of an escape event.

When there is a single equilibrium–that is, when $\beta_2^*(m)$ does not exist–inflation expectations are always stable. This is direct when the unique equilibrium is $\beta_1^*(m)$. For high deficit-mean levels, the stable level of expected inflation may be $\beta_3^*(m)$, and it can imply very high inflationary expectations, blurring the difference between explosive and stable behavior. In other words, expected inflation may be growing toward a very high level of several thousand percent in annualized terms, which is not infinity, but it makes very little difference in practice. Moreover, we cannot say that this is unstable behavior, as the very high level of equilibrium expected inflation at $\beta_3^*(m)$ is a stable equilibrium. In any case, when the low SCE $\beta_1^*(m)$ does not exist, we use the value π_{max}^* in our numerical calculations.

The policy significance of these levels is that equilibrium inflation depends nonlinearly on the deficit state. As we increase the deficit-mean from d_5

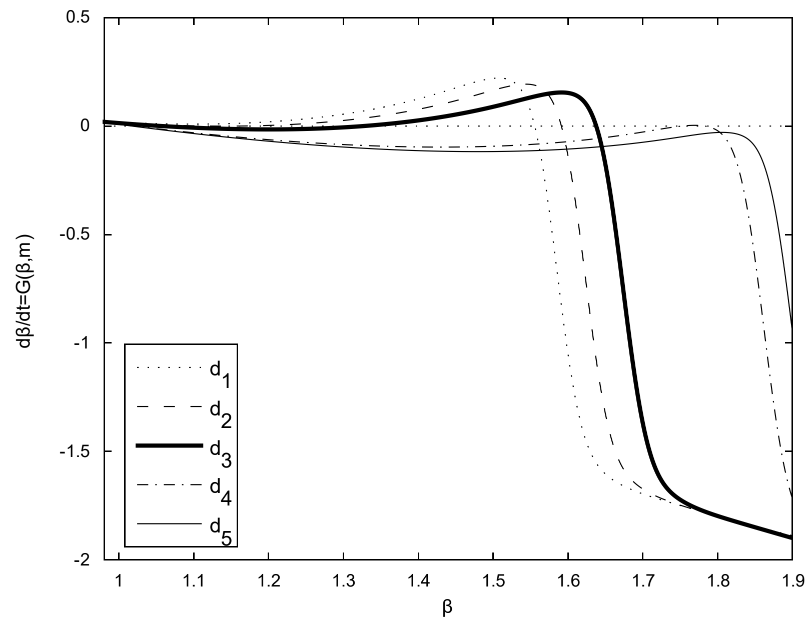

Figure 3b: **Deficits and Conditional Self-Confirming Equilibria**. Note: This figure shows that, for mean deficit levels on or above a threshold, here d_4, the model has a stable conditional SCE at a very high inflation level. Counterintuitively, for all mean deficit levels above d_4, this high equilibrium increases, not decreases, as the deficit-mean decreases. See the main text for the parameter values employed to elaborate figures 3a and 3b. Source: Own calculations.

to d_1, equilibrium inflation experiences a big jump whenever there is an escape event or if $d > d_4$, as the economy moves suddenly from a low to a very high-inflation equilibrium. When the deficit increases faster, this will happen sooner, as the domain of attraction of $\beta_3^*(m)$ expands as the deficit-mean increases. Once in the domain of attraction of $\beta_3^*(m)$, however, small reductions in d will cause an increase, not a decrease in the level of equilibrium inflation.

To ensure that the economy finds itself in the domain of attraction of a low-inflation equilibrium, it is a necessary condition to maximize the chance of the success of a stabilization program to reduce the deficit level from d_1 to a level $d < d_4$. One can't reduce inflation by progressively reducing the deficit from d_1 to levels higher than d_4, unless the economy experiments a shock that accidentally takes inflation expectations to the domain of attraction of the low-level SCE.

At the beginning of such a program, inflation will not fall, and it may even become higher. The necessary reduction in the deficit-mean level has to be done at once, not progressively. Thus, this model proposes that a fiscal reform is a necessary condition to reduce inflation, as in SWZ; in addition, we find that this reform must be strong enough if the economy starts from the domain of attraction of a high-inflation equilibrium, as it often has happened in the Latin American monetary history.

Although our SCEs can display more general features, when $d_4 < d < d_2$, they retain some of the logic of the SSEs in Bruno and Fischer (1990), if we consider just $\beta_1^*(m)$ and $\beta_2^*(m)$. For instance, their stability depends on the formation mechanism of inflation expectations. Of course, these models have some key differences. Bruno and Fischer's 1990 model does not feature regimes in the deficit. Nonetheless, exogenous variations in its deficit affect their dual inflation equilibria in a similar way to a mean deficit regime switch in SWZ and in our model.

As mentioned, the definition of SCE applies for each m-state and determines the stability regions of inflation expectations. Whereas the agent forms its inflation beliefs following the rule in equation (6), the SCEs represent the average dynamics of such expectations as $\nu \to 0$. They will be close to the actual dynamics when the deficit state m is highly persistent. Following SWZ, we now define an unconditional SCE.

Definition 2. Unconditional self-confirming equilibrium (SCE). An unconditional SCE is a probability distribution over inflation histories $\beta^T \equiv \{\pi_1, \pi_2, ..., \pi_T\}$ and a β such that

$$\mathbb{E}[\pi_t] - \beta = 0. \tag{29}$$

This equilibrium is found by finding the zero(s) of the following function:

$$\begin{aligned}
\hat{G}(\beta) \equiv & E[\tilde{g}(\pi_t^*, \beta, d_t(m_t, \varsigma_t, d_{t-1}))] \\
= & \sum_{k=1}^{h} \left[\theta\lambda(\beta)\Psi_{[k]}(\beta, \tilde{\omega}(\beta))\right] \bar{q}_k \\
& + \sum_{k=1}^{h} \bar{\pi}_1^*(k) \left[1 - \Phi\left(\frac{\log(\tilde{\omega}(\beta) - \log(\bar{d}(k))}{\sigma_d(k, \bar{d}(k))}\right)\right] \bar{q}_k - \beta,
\end{aligned}$$

where $\bar{q}_k$ is the ergodic distribution of the joint state k. Since $\hat{G}(\beta)$ is an average over m of $\hat{G}(\beta, m)$, we can expect the equilibria to behave as in a hypothetical average state m. Thus, in view of Proposition 1, there exists at least one unconditional SCE and up to three unconditional SCEs.

6 Escapes and Reforms

We will proceed using the definition of SCE that fixes m, i.e., with a fixed deficit-mean level. This will usually be the most useful case for practical purposes because we want to evaluate the probable path of the economy given the current policy stance. The following definitions follow SWZ:

Definition 3. A *reform* is called for when, without it, conditions (9) and (10) would be violated as long as the regime state m remains constant.

Definition 4. A *fundamental reform* takes place when, under its implementation, the state m switches to satisfy conditions (9) and (10).

Definition 5. A *cosmetic reform* occurs when a reform is called for, the current state m remains the same, and inflation is reset. Such a reset occurs by setting inflation to the inflation's low deterministic SSE value $\pi_1^*(m_t)$ plus some noise:

$$\pi_t^* = \pi_1^*(m_t) + \varepsilon_\pi, \tag{30}$$

where ε_π has the following probability density:

$$p_\pi(\varepsilon_\pi|\tilde{m}_t) = \frac{\exp\{-\log[\pi_1^*(m_t) + \varepsilon_\pi] - \log \pi_1^*(m_t)]^2/2\sigma_\pi^2\}}{\sqrt{2\pi}\sigma_\pi[\pi_1^*(m_t) + \varepsilon_\pi]\Phi[(-\log\delta - \log[\pi_1^*(m_t)])/\sigma_\pi}, \tag{31}$$

if $-\pi_1^*(m_t) < \varepsilon_\pi < 1/\delta - \pi_1^*(m_t)$, and $p_\pi(\varepsilon_\pi|\tilde{m}_t) = 0$ in all other cases. These last two inequalities ensure that inflation is reset to a level that satisfies (9) and (10).[17]

As the model has up to two stable equilibria, it is an important event when the economy switches from the domain of attraction of the low to that of the high SCE. Our definition of escape departs from SWZ:

Definition 6. An escape takes place when inflation beliefs fall outside the domain of attraction of the low and stable SCE, $\beta_1^*(m)$, and inside the domain of attraction of $\beta_3^*(m)$, the high SCE. This is the case if $\beta_t > \beta_2^*(m)$ whenever $\beta_2^*(m)$ exists.

If there is just one SCE, either $\beta_1^*(m)$ or $\beta_3^*(m)$, there are no escapes in the sense of SWZ, but we can compute escape probabilities depending on the relevant domain of attraction, according to Definition 6; thus, we can define escape probabilities by taking these situations as special cases, as we do below.

For the given β_t and β_{t-1}, we now consider shocks on the deficit that would contribute to an escape event. Consider, then, again that $d(s_t, d_{t-1}) = \bar{d}(m_t) + \varepsilon_d(\varsigma_t, d_{t-1})$. We then have $\underline{\omega}_t(m_t, \varsigma_t)$, defined as the value of $\varepsilon_d(\varsigma_t, d_{t-1})$, such that $\pi_t = \beta_2^*(m)$, and $\overline{\omega}_t(m_t, \varsigma_t)$, defined as the value of $\varepsilon_d(\varsigma_t, d_{t-1})$ such that $\pi_t = \delta^{-1}$ (i.e., its upper bound). Crucially, such inflation realizations would drive inflation expectations toward their domain of attraction of $\beta_3^*(m)$. One can prove that their values are as follows:[18]

$$\underline{\omega}_t(m_t, \varsigma_t) = \frac{1}{\gamma}(\lambda(\beta_t) - \theta\lambda(\beta_{t-1})\beta_2^*(m_t)^{-1}) - \bar{d}(m_t)$$

$$\overline{\omega}_t(m_t, \varsigma_t) = \frac{1}{\gamma}(\lambda(\beta_t) - \delta\theta\lambda(\beta_{t-1})) - \bar{d}(m_t).$$

Conditional on the regime state that the model is in during period $t-1$, the probability of an escape-provoking event, when $\beta_2^*(m)$ exists, is

$$\begin{aligned} Pr\{\underline{\omega}_t(m_t,\varsigma) < \varepsilon_d(\varsigma_t, d_{t-1}) < \overline{\omega}_t(m_t, \varsigma)|s_t = s, d_{t-1} = d'\} \\ = F_d(\overline{\omega}_t(m_t, \varsigma_t)|s_t = s, d_{t-1} = d') - F_d(\underline{\omega}_t(m_t, \varsigma_t)|s_t = s, d_{t-1} = d'). \end{aligned}$$

[17]Note that $\pi_t^* = \pi_1^*(m_t) + \varepsilon_\pi > 0$ if and only if $\varepsilon_\pi > -\pi_1^*(m_t)$. Moreover, if $\varepsilon_\pi < 1/\delta - \pi_1^*(m_t)$, then $\pi_t^* = \varepsilon_\pi + \pi_1^*(m_t) < \delta^{-1}$.

[18]The problem is to find the value of $\varepsilon_d(\varsigma_t, d_{t-1})$, such that $\pi_t = x$ for the given level of expectations β_t and β_{t-1}. Consider then equation (8), with $d(s_t, d_{t-1}) = \bar{d}(m_t) + \varepsilon_d(\varsigma_t, d_{t-1})$.

This is the probability of having a shock to the deficit that would lead inflation to be greater than $\beta_2^*(m)$ but smaller than δ^{-1}. This shock may have idiosyncratic or external provenance. In general, the econometrician does not observe the regime state that the economy is in during period t, and the escape-provoking event probability is

$$\sum_{s_0=1}^{h} Pr(s_t = s_0|\pi^{t-1}, \phi) \left[F_d(\overline{\omega}_t(m_0, \varsigma_0)|s_0, d_0) - F_d(\underline{\omega}_t(m_0, \varsigma_0)|s_0, d_0)\right].$$

Under unique SCEs, there are no escapes in the sense of SWZ, as mentioned, but note that if $\beta_3^*(m)$ is the unique equilibrium, then the economy is permanently in the domain of attraction of the high SCE; therefore, the probability of falling into the domain of the very high SCE, for those states m such that $\beta_3^*(m)$ is the unique equilibrium, as

$$Pr\left(\underline{\omega}_t(m_t, \varsigma_t) < \varepsilon_d(\varsigma_t, d_{t-1}) < \overline{\omega}_t(m_t, \varsigma_t)|s_t = s, d_{t-1} = d'\right) = 1,$$

likewise, if m is such that $\beta_1^*(m)$ is the unique equilibrium, the probability of falling into the domain of the very high SCE is

$$Pr\left(\underline{\omega}_t(m_t, \varsigma_t) < \varepsilon_d(\varsigma_t, d_{t-1}) < \overline{\omega}_t(m_t, \varsigma_t)|s_t = s, d_{t-1} = d'\right) = 0,$$

because the economy is permanently in the domain of attraction of the low SCE.

We can also compute the probability of a high inflation at each point in time. For a given level of inflation π^H regarded as "high", we can compute

$$\begin{aligned}
\underline{\omega}_t^H(m_t, \varsigma_t) &= \frac{1}{\gamma}\left(\lambda(\beta_t) - \theta\lambda(\beta_{t-1})/\pi^H\right) - \bar{d}(m_t) \\
\overline{\omega}_t^H(m_t, \varsigma_t) &= \frac{1}{\gamma}\left(\lambda(\beta_t) - \delta\theta\lambda(\beta_{t-1})\right) - \bar{d}(m_t).
\end{aligned}$$

Then, the probability of observing "explosive" inflation, i.e., inflation above the high level π^H, is

$$
\begin{aligned}
&Pr(\pi_t > \pi^H | s_t = s, d_{t-1} = d') \\
&\qquad =Pr\left(\underline{\omega}_t^H(m_t, \varsigma_t) < \varepsilon_d(\varsigma_t, d_{t-1}) < \overline{\omega}_t^H | s_t = s, d_{t-1} = d'\right) \\
&\qquad =F_d(\overline{\omega}_t^H(m_t, \varsigma_t) | s_t = s, d_{t-1} = d') \\
&\qquad\quad - F_d(\underline{\omega}_t^H(m_t, \varsigma_t) | s_t = s, d_{t-1} = d'),
\end{aligned}
$$

conditional on the regime state the model is in during period $t-1$, as well as the latest deficit level.

7 Data

We use inflation as measured by the month-to-month changes in seasonally adjusted consumer price index from the National Statistics Institute (Instituto Nacional de Estadística y Geografía, INEGI). Our dataset comprises the period starting in February 1969 and ending in July 2019 (figures 7 and 8).

Our estimation produces an implied seigniorage-financed deficit series, obtained from equilibrium condition (8). A natural exercise, then, is to compare such series with several measures of the fiscal deficit. To do so, we use the economic balance series, a common measure of the fiscal deficit.[19] In addition, we use the Public Sector Borrowing Requirements (PSBR) series, a much broader measure of the fiscal deficit. Yet, the economic balance series has a longer history than that of the PSBR. We use these series with an annual frequency.

In particular, the economic balance series is from 1977 to 2019. Because the estimation methodology for the PSBR was changed, we concatenate the growth rates of these two series when available. The first, using the former methodology, is available for the 1990–2014 period, and the second, using the more recent one, is available for the 2000–2019 period.[20] It is worth

[19]The economic balance (also known in Mexico as the traditional balance) equals the government's revenues minus its expenses. The revenues include tax collection, social security fees and rights, revenue from financing entailing the sale of goods and services, and financial products and recovery value from sales of fixed assets, among others. The expenses category includes those needed for public sector operation such as personnel services payments, materials, supplies and general services, capital accumulation, public debt service, subsidies, and transfers to the private and social sectors (SHCP, 2014).

[20]Such series are from the Mexican Ministry of Finance (*Secretaría de Hacienda y Crédito Público*, SHCP).

reemphasizing that we use these series only for comparison purposes and never as part of the estimation.

8 Model Estimation

The model has three key variables: inflation, inflation expectations, and seigniorage-financed deficits. Inflation is the only input variable with which we estimate the model. The mean and variance regime states are unobservable to the econometrician, but we can estimate the probability of being in a certain regime state in a given period t. We interpret the state with the highest probability in a given period as indicative of the regime state prevalent in the economy in that period, although the difference with the next highest probability is small in some periods.

We estimate the parameters ν, λ_0, λ_1, γ, δ, θ, and ϑ from the data. The parameter γ is invariant to a re-normalization of $\bar{d}(m_t)$ and $\sigma_d^2(\varsigma_t)$ by some constant.[21] Without loss of generality, we set $\gamma = 1$ and $\delta = 0.01$. We assume that $\theta = 0.99$. These values are in line with those used in SWZ. One key aspect of the model is the number of deficit and volatility regime states which are quantities that are fixed before the estimation. To determine them, we use the Schwarz Criterion, a.k.a. Bayesian Information Criterion (BIC), defined as BIC$= L(\hat{\phi}) - \log(T)k/2$, where $L(\hat{\phi})$ is the maximized log-likelihood, T is the number of observations, and k is the number of estimated parameters. The results are reported in Table 1.

We found that the best model, according to the BIC, has six regimes for m and two for ς. We also calculated seven other criteria: Akaike Information Criterion (AIC), Hannan-Quinn, Generalized Cross-Validation, Mallows' C_p, Amemiya's PC, and Hocking's S_p. All criteria agree on the number of states of the deficit volatility, but Hannan-Quinn suggests using one more state for the deficit-mean, whereas the remaining criteria regard eight or more as a better choice for the number of deficit-mean states.

As our objective is to interpret Mexican historical inflation data, we found that six states for the deficit-mean are sufficient to identify historical changes in the fiscal and monetary authorities' policy stances. As we increase the number of deficit-mean states, we find either that some periods are better represented by a new deficit-mean or that the latter is redundant in the

[21] It determines a standardization of the price level and the nominal money stocks.

		BIC$= L(\hat{\phi}) - \log(T)k/2$
$n_m = 6$	$n_\varsigma = 2$	2498.42
$n_m = 6$	$n_\varsigma = 1$	2439.85
$n_m = 6$	$n_\varsigma = 3$	2449.32
$n_m = 5$	$n_\varsigma = 2$	2494.97
$n_m = 7$	$n_\varsigma = 2$	2497.80

Table 1: **Model Selection**. We used the BIC to choose the optimal number of states. $L(\hat{\phi})$ is the maximized log-likelihood, T is the number of observations, k is the number of estimated parameters, n_m is the number of states for the mean deficit, and n_ς is the number of states for the deficit volatility. On the top row, we display the optimal number of states. Then, we show the BIC calculated for small changes in the number of states. We calculated the BIC for several other combinations of the number of states, always obtaining lower values. We also tried six other model selection criteria: AIC, Hannan-Quinn, Generalized Cross-Validation, Mallows' C_p, Amemiya's PC, and Hocking's S_p, obtaining the same number of states for the volatility but at least one more state for the mean deficit. However, additional state(s) do not significantly change our interpretation of the historical data. Source: Own estimations with data from INEGI.

sense that it attempts to provide a marginal refinement of an already identified policy stance. After $n_m = 6$, the new deficit-mean state tends not to be noticeably different from a previously estimated one, and although it improves the fit of the model, it does so marginally and does not contribute significantly to the discussion on the economic content of the model. In sum, the number of states reaches a reasonable compromise between the competing goals of the model's parsimoniousness and fit.

In the case of the 6×2 model, we impose the following restrictions on the transition matrices, following Sims, Waggoner, and Zha (2008) and SWZ:

$$Q_m = \begin{pmatrix} p_{11} & 1-p_{11} & 0 & 0 & 0 & 0 \\ \frac{1-p_{22}}{2} & p_{22} & \frac{1-p_{22}}{2} & 0 & 0 & 0 \\ 0 & \frac{1-p_{33}}{2} & p_{33} & \frac{1-p_{33}}{2} & 0 & 0 \\ 0 & 0 & \frac{1-p_{44}}{2} & p_{44} & \frac{1-p_{44}}{2} & 0 \\ 0 & 0 & 0 & \frac{1-p_{33}}{2} & p_{55} & \frac{1-p_{55}}{2} \\ 0 & 0 & 0 & 0 & 1-p_{66} & p_{66} \end{pmatrix},$$

$$Q_\varsigma = \begin{pmatrix} q_{11} & 1-q_{11} \\ 1-q_{22} & q_{22} \end{pmatrix}.$$

These restrictions have some implications for the Markov chains associated with the deficit-mean and the deficit-volatility. The first is that any switch between regimes must go through the intermediate regime state. For example, to switch from the first to the third state, the state must go through the second one. The second implication is that if the Markov chain is at some point in a regime state different from the first or sixth, it has an equal probability of switching to the adjacent regime states. The implication is that less parameters are needed to describe the model, simplifying the estimation of the model.[22]

We estimate the model by solving the following problem:

$$\max_{\phi} p(\pi^T|\phi),$$

where $p(\pi^T|\phi)$ is the inflation's likelihood function resulting from the model. We present its derivation in the Appendix. Specifically, the parameter vector is $\phi = (\nu, \lambda, \vartheta, \bar{d}_{\{i\}}), \sigma_{\{j\}}, p_{\{i,i\}}, q_{\{j,j\}}, \sigma_\pi)$, where $i = 1, ..., 6$ and $j = 1, 2$.

[22]We impose bounds on $p_{i,i}$ and $q_{j,j}$. Being probabilities, they must be greater than zero and less than one, where $i = 1, ..., 6$, and $j = 1$ and 2.

Parameter	Estimate	Standard Errors
ν	0.014	0.001
λ_0	0.178	0.105
λ_1	29.27	2.071
ϑ	0.702	0.288
$\bar{d}_1$	0.0062	0.003
$\bar{d}_2$	0.0044	0.004
$\bar{d}_3$	0.0035	0.002
$\bar{d}_4$	0.0028	0.002
$\bar{d}_5$	0.0023	0.001
$\bar{d}_6$	0.0021	0.001
σ_1	1.904	1.680
σ_2	0.666	0.580
$p_{1,1}$	0.87	0.065
$p_{2,2}$	0.90	0.062
$p_{3,3}$	0.84	0.056
$p_{4,4}$	0.87	0.044
$p_{5,5}$	0.88	0.045
$p_{6,6}$	0.97	0.019
$q_{1,1}$	0.71	0.070
$q_{2,2}$	0.90	0.029
σ_π	0.03	3.812

Table 2: **Parameter Estimates**. Note: All parameters are significant at the usual confidence levels, with the following exceptions. We find that σ_π is not statistically significant. The volatility regime parameters σ_1 and σ_2 are statistically significant at the 85% confidence level. The maximized log-likelihood is 2565.688. The estimation sample comprises February 1969 to July 2019. Source: Own estimations.

To estimate the model's parameters, we implement the maximum likelihood method with a numerical optimization algorithm that combines the Nelder-Mead with the quasi-Newton algorithm.[23] Estimated parameter standard errors are obtained from the Hessian matrix of the likelihood function using the Cramér-Rao bound.[24] Table 2 presents our estimates $\hat{\phi}$ and their corresponding standard errors.

We obtain a small ν implying that agents form their inflation expectations by allocating more weight to their previous inflation beliefs, relative to their past errors–that is, deviations between realized inflation and beliefs.

The transition probability estimates $(p_{i,i})$ indicate persistent regimes for the deficit-mean states. In the case of the regime for the variance deficit $(q_{j,j})$, both states are persistent as well. Nonetheless, the high variance regime state (σ_1) is not as persistent as the low variance one, as $q_{2,2} > q_{1,1}$. Taken together, the small value of ν and the regime states' persistence (i.e., estimates of $p_{i,i}$ and $q_{j,j}$ being close to one) imply that SCEs are close to the SSEs, as previously explained.

In the model, a fundamental reform is associated with a transition from a higher to a lower mean regime state. Thus, one can think of the regime states' persistence as reflecting a friction in switching regimes. In practice, implementing a reform is, in general, costly. Thus, one can see their persistence as an advantage. In effect, if the economy were in a low deficit-mean regime state, it would be costly to switch out of it.

Table 3 presents the stationary or unconditional probabilities for all regime states. Roughly, they capture the fraction of time the economy spends in each regime during the sample period. The unconditional probability of the low-variance regime is relatively high. This is because the economy seems to switch to the high-variance regime only occasionally, when there is a shock too high to be explained either by the deficit-mean state alone or by the low regime deficit volatility. As the deficit and the volatility states are assumed to be independent, the joint states' unconditional probabilities are just the

[23]The first algorithm is applied in the first stage to obtain a robust first solution. This intermediate solution is then refined for accuracy with the second algorithm, obtaining a final solution. Nevertheless, the optimal estimates are still sensitive to the initial values of ϕ. Hence, we build a grid to start the optimization at many initial points. The optimization program iterates on such a grid, searching for a final estimate. We verify that the final estimate is associated with a real, invertible, and negative definite Hessian matrix of the likelihood function.

[24]It is also known as Frechet-Darmois-Cramér-Rao inequality.

Deficit-Mean Regime (Unconditional Probabilities)	
$m = 1$	0.15
$m = 2$	0.10
$m = 3$	0.13
$m = 4$	0.15
$m = 5$	0.17
$m = 6$	0.29
Deficit-Variance Regime (Unconditional Probabilities)	
High ($\varsigma = 1$)	0.25
Low ($\varsigma = 2$)	0.75

Table 3: **Stationary Markov Regimes Probability Estimates**. Note: By independence between the mean and variance regime states, the joint states' stationary probabilities are just the product of the deficit-mean and variance marginal probabilities. Probabilities may not sum to 1 due to rounding. Sources: Own estimations with data from INEGI.

product of the probabilities of the mean and volatility states.

We have estimated probabilities of the deficit-mean state that are conditional on inflation history up to period $t-1$, i.e., one period before the current one, t (figure 4). This timing convention is in line with the escape-provoking probability. We obtain these probabilities by adding across different variance regime states, keeping the mean state fixed.

Inflation Expectations and SCEs

We have mentioned our interest in understanding events in which inflation expectations surpass a level after which they become explosive. SWZ define such a situation as an escape event. As described therein, their model has generally two conditional SCEs for each m-state: a low- and a high-inflation expectation equilibrium. While the low SCE is a stable fixed point, the high SCE is unstable. We have computed the function $\hat{G}(\beta, m)$ with Monte Carlo integration and then obtained the SCEs by locating the zeros of that function. In our model, we have up to three SCEs, and the first two have similar properties to those in SWZ. In addition, we sometimes obtain a stable SCE with very high inflation. As explained above, we define an escape event as the probability of falling into the domain of attraction of the SCE with very high inflation. We present our estimations of deterministic and self-

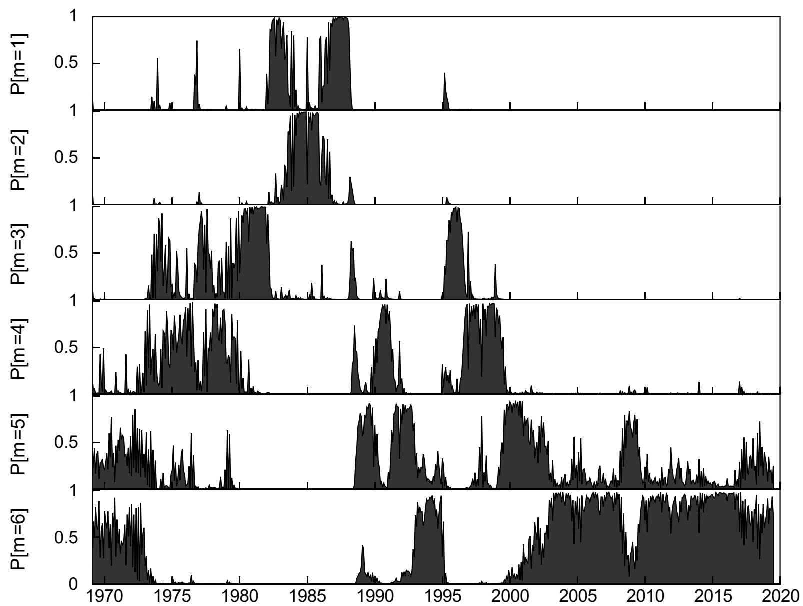

Figure 4: **Probabilities of being in each deficit-mean regime, conditional on the information in period *t-1***. Note: $m_t = 1$ denotes the highest deficit-mean state, and $m_t = 6$ the lowest deficit-mean state, respectively. As the deficit-mean state is not observable, this figure depicts the estimated probabilities of being in each state at each period. Source: Own estimations with data from INEGI.

confirming equilibria in Table 4 and figure 5 for the six m-states.

All m-states have at least one fixed-m SCE, as predicted by Proposition 1, and in the case of the second-largest mean deficit level, there are three equilibria. Note that, for the high deficit state, the stable equilibrium is the very high-inflation SCE and implies annualized rates of more than $6,000\%$ (Table 4 and figure 5). For that reason, if the deficit-mean regime switches to such a state, then the inflation expectations turn explosive, notwithstanding the existence of a stable equilibrium. Thus, unless a reform takes place, inflation expectations will significantly and unceasingly amplify until the SCE is reached. It is also surprising that a conditional SCE exists even when its corresponding deterministic steady-state equilibrium does not. The reason is that the definition of SSE effectively caps steady-state equilibria at the level $\pi^*_{\max}$, thereby ruling out "explosive" steady-states.

Deterministic Equilibria (SSE)	
$\pi_1^*(1), \pi_2^*(1)$	π^*_{max}, n.a.
$\pi_1^*(2), \pi_2^*(2)$	$1.0803, 1.2550$
$\pi_1^*(3), \pi_2^*(3)$	$1.0258, 1.6558$
$\pi_1^*(4), \pi_2^*(4)$	$1.0108, 2.1405$
$\pi_1^*(5), \pi_2^*(5)$	$1.0049, 2.5914$
$\pi_1^*(6), \pi_2^*(6)$	$1.0029, 2.8380$
π^*_{max}	1.1447
Unconditional SCE	
$\pi_1^*, \pi_2^*, \pi_3^*$	1.0214, n.a., n.a.
Conditional fixed-m, SCE	
$\pi_1^*(1), \pi_2^*(1), \pi_3^*(1)$	π^*_{max}, n.a., 1.4236
$\pi_1^*(2), \pi_2^*(2), \pi_3^*(2)$	$1.0962, 1.2055, 1.5899$
$\pi_1^*(3), \pi_2^*(3), \pi_3^*(3)$	1.0271, n.a., n.a.
$\pi_1^*(4), \pi_2^*(4), \pi_3^*(4)$	1.0112, n.a., n.a.
$\pi_1^*(5), \pi_2^*(5), \pi_3^*(5)$	1.0052, n.a., n.a.
$\pi_1^*(6), \pi_2^*(6), \pi_3^*(6)$	1.0031, n.a., n.a.

Table 4: **Deterministic and Self-Confirming Equilibria**. Note: We report the equilibria found numerically. The value π^*_{max} is imputed when a low SSE or low SCE does not exist. Otherwise nonexistent equilibria are denoted as "n.a." Sources: Own estimations with data from INEGI.

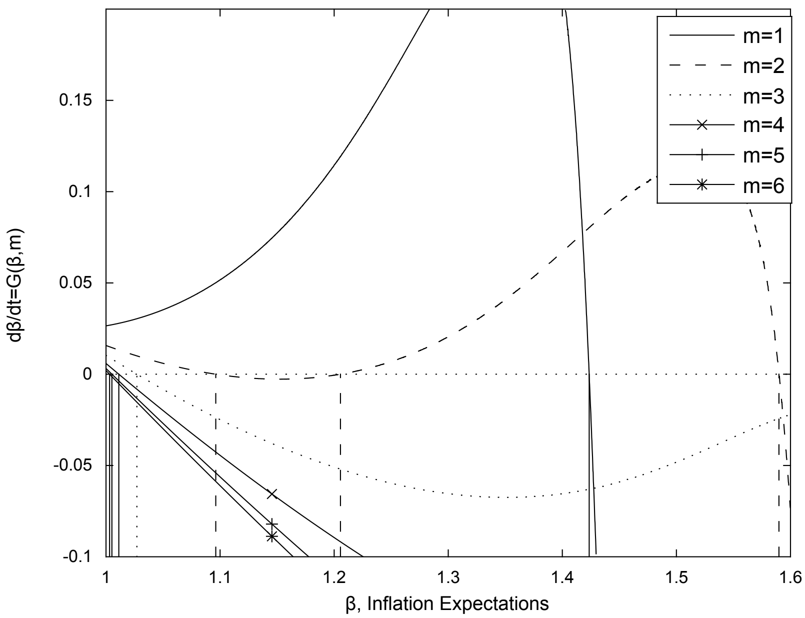

Figure 5: **Conditional fixed-m SCEs**. Note: Each conditional SCE is determined when the function $G(\beta, m)$ crosses the value of 0 for each deficit-mean state m. Because the equation is in continuous time, SCEs are determined when expectations do not change–i.e. $\hat{\beta} = 0$, or $d\beta/dt = 0$. The highest and lowest deficit-mean regimes are $m = 1$, and $m = 6$, respectively. For the estimated model, each state has only one conditional SCE, except state $m = 2$, which has three conditional SCEs. Source: Own estimations with data from INEGI.

9 General Discussion

In this section, we show that the model's dynamics match with key economic events quite well. To do this, we relate the model's regime changes in seigniorage and equilibrium behavior to past historical episodes in Mexican fiscal policy. Importantly, we show that our model clearly captures quantitatively the presence or absence of a close relationship between fiscal and monetary policy and that it does so better than alternative models, such as the one in SWZ. We conclude that an adequate money-demand specification is important not only from a descriptive point of view, but also because it gives distinct policy recommendations and has allowed us to make a stronger case for the historical role of fiscal dominance in the origins of Mexican inflation during the last three decades of the past century, with one caveat. There is sometimes a delay between historical increases in the deficit and the increase in model-implied seigniorage-financed deficits, suggesting that deficits were initially financed by debt, which was eventually paid with seigniorage. Conversely, reducing spending did not immediately reduce seigniorage, while preexisting overdue debt payments were being honored with seigniorage. This seemed to be a common occurrence during the fiscal dominance period. We mostly base our economic narrative on Ortiz (1991), Musacchio (2012), Ramos-Francia (1994), Sidaoui (2000), and Whitt (1996).[25]

Fiscal policy in Mexico became expansive starting roughly in the year 1970. The resulting deficits were financed partially with debt but also with seigniorage, and inflation followed afterward. Several stabilization attempts were conducted, and the most important occurred in 1995, which marked the end of fiscal dominance and the beginning of lasting monetary stability in Mexico, although the 1988 stabilization program was also quite successful.

In what follows, we compare these events with the data produced by the model, which we summarize with figures 6 to 10. We first consider the mean deficit states with the highest probability, and inflation (figure 6). We see

[25] We have two remarks. First, we base some of our claims on probabilistic statements. However, for simplicity, we are not always explicit about such claims. Thus, we might state that the regime transitions to the low mean deficit state, meaning that the probability of being in such a regime state is higher than the probability of being in all other states. Second, the model deficit levels are not directly comparable with the deficit data levels for the following reasons: the model deficits are subject to a rescaling by a factor of $1/\gamma$ and $\bar{d}(m_t)$, and given the deficit distribution, the model does not allow for negative deficit levels. In short, the data capture the general deficit, and the model only accounts for the seigniorage-financed portion of the deficit.

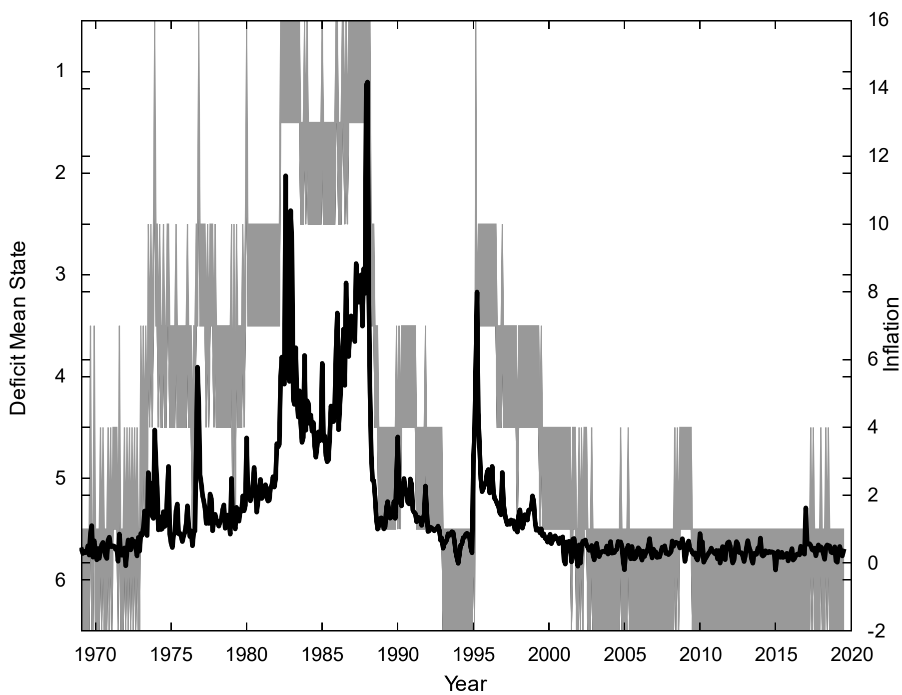

Figure 6: **Highest-Probability Deficit-Mean Regimes, Conditional on the Information up to the Previous Period, and Inflation**. Note: We show the most likely deficit-mean state, conditional on the information in period $t-1$. Thus, the left-hand scale ranges from 1 to 6. Monthly inflation is depicted on the right-hand scale. Source: Estimations with data from INEGI.

that inflation increases as we switch to a higher deficit-mean regime, and vice versa. The reduction of inflation in the early nineties to previous decades' levels was accomplished by a fundamental reform that took the deficit-mean regime to a level even lower than the mean deficit level that produced roughly the same inflation before. We also compare inflation, inflation expectations, and SCEs (figure 7), escape probabilities (figure 8), actual and model-implied deficits (figure 9), and model-implied deficits and deficit-mean states (figure 10).

Prior to 1970, the fiscal and monetary policies were successful in maintaining price stability for several years. Government expenses swiftly adjusted to unanticipated changes in public revenues. At the time, the monetary base growth was under strict control. Thus, inflation maintained a low and stable level. Commercial banks' reserves were a noninflationary source of government finance (Ramos-Francia, 1994). In line with these events, the low mean deficit regime was predominantly the most probable, and inflation expectations remained very near to their corresponding low SCE.

Economic policy considerably changed after Echeverría became president in 1970.[26] Government expenses, heavily financed through seigniorage, substantially increased. As a result, the monetary base growth rose. In figure 6, we can see that during this administration, the deficit-mean level progressively increased after 1970 from the lowest state $m = 6$, to state $m = 3$, albeit with occasional and temporary intermediate switches that could be explained by noise. There was also a brief spike to the highest deficit-mean regime (figures 6 and 10). Finally, this is the most volatile period in our entire sample.

Inflation expectations were clearly lower than their equilibrium levels and started a continuous increase (figure 7). Additionally, during this time, as bank reserves fell, their use as a source of public financing decreased. High fiscal deficits accompanied high current account deficits, in line with the twin deficits hypothesis. In 1976, a balance-of-payments crisis took place and the government devalued the peso, ending a fixed exchange-rate regime of more than 20 years.

At the beginning of the López-Portillo administration (1976–1982), an IMF-backed stabilization program was implemented and initially considered

[26]Mexico's presidents for the 1970–2018 period were as follows: Echeverría Álvarez, 1970–1976; López Portillo, 1976–1982; de la Madrid Hurtado, 1982–1988; Salinas de Gortari, 1988–1994; Zedillo Ponce de León, 1994–2000; Fox Quezada, 2000–2006; Calderón Hinojosa, 2006–2012; Peña Nieto, 2012–2018; and López Obrador, 2018-2024.

a success. This success was short-lived: discipline on public finances was soon to be neglected again, due to optimism produced by a newly discovered supergiant oil field.[27] This was followed by a period during which the government kept its distance from the Fund (IMF, 2001). Government expenditures increased to unprecedented levels.

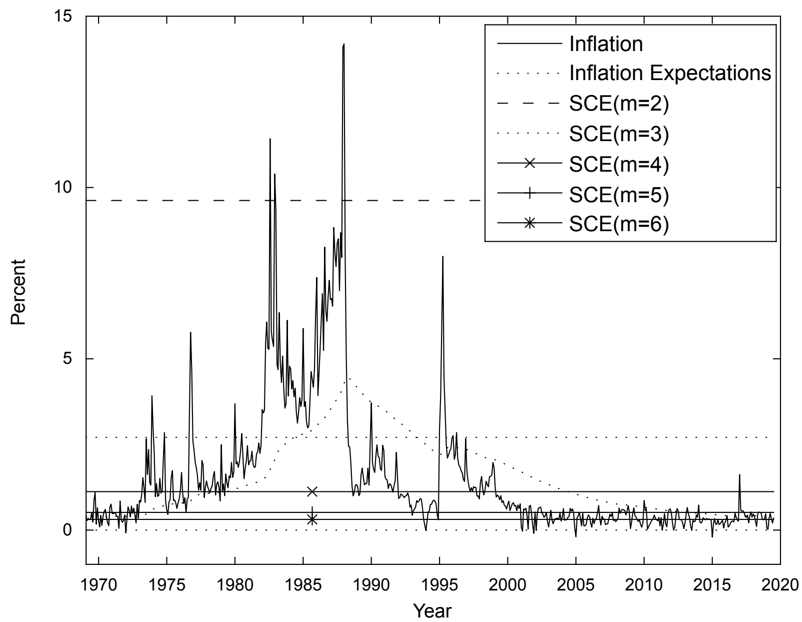

Figure 7: **Monthly Inflation, Inflation Expectations, and Conditional SCEs**. Note: Monthly inflation is the month-to-month (m-m) percentage change of the seasonally adjusted CPI. Conditional SCEs are depicted by horizontal dotted lines. The lowest and highest mean deficit regimes are $m = 6$ and $m = 1$, respectively. There are some conditional SCEs outside this figure. Source: Own estimations with data from INEGI.

The fiscal and current account deficits increased concomitantly. The Mexican external debt grew substantially (see figures 11 and 12). Consistently

[27]A drawing of just SDR 100 million was made in February 1977 (out of SDR 618 million available). The government was able to meet its external financing requirements through commercial banks, due partly to the adjustment program and partly to the new oil reserves (IMF, 2001).

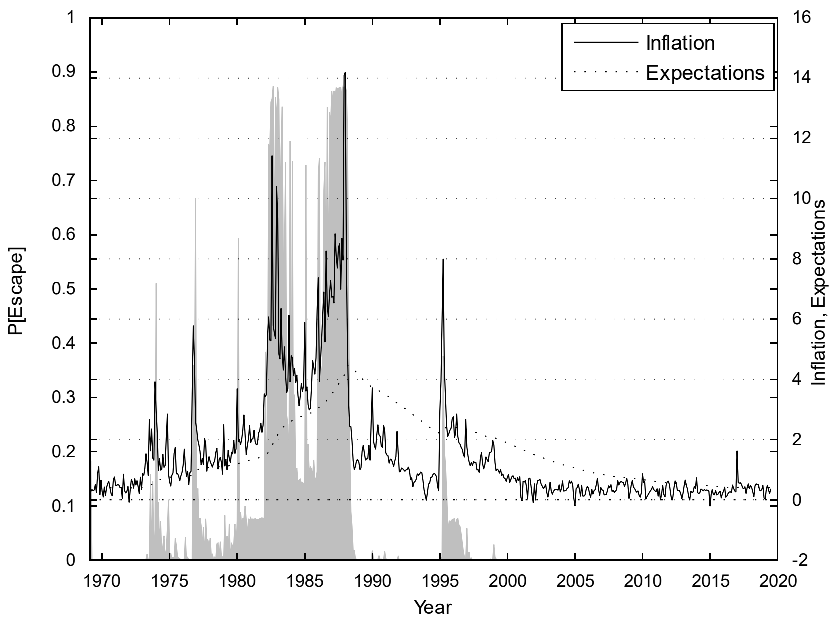

Figure 8: **Escape Probabilities and Inflation Expectations**. Note: This figure overlays the monthly inflation and inflation expectations on escape probabilities–i.e., the probability of falling within the domain of attraction of the equilibrium with very high inflation during the next month. Source: Own estimations with data from INEGI.

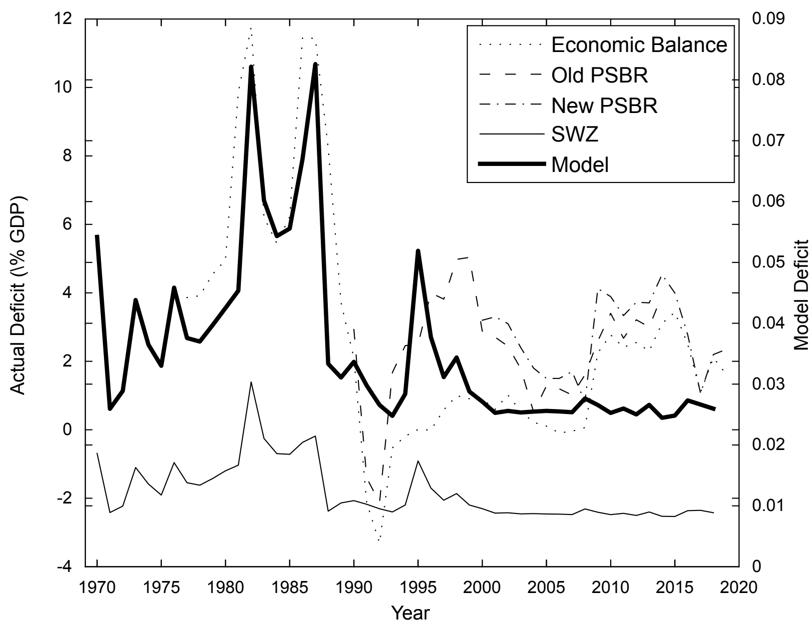

Figure 9: **Deficits and Model-Implied Deficits**. Note: There is a close association between model-implied deficits and two measures of the Mexican public sector deficit up to 1995. Afterwards, there is no clear association. We also show the deficits implied by the model in SWZ. To obtain annual estimates, we calculate the sum of the monthly model-implied deficits over each year. Source: Own estimations with data from INEGI and SCHP.

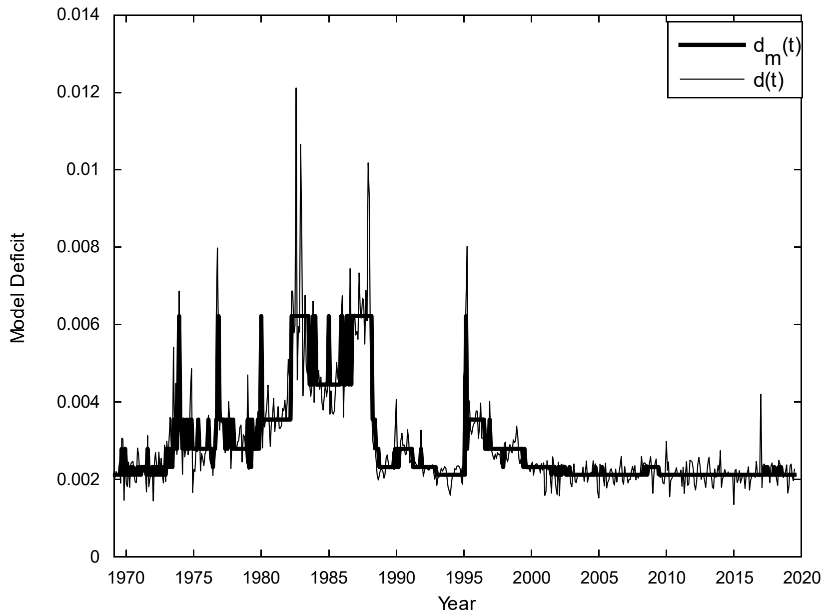

Figure 10: **Deficit-Mean Regimes and Model-Implied Deficits**. Note: We overlay the model-implied deficits on the deficit-mean of the state most likely at each point in time. Source: Own estimations with data from INEGI.

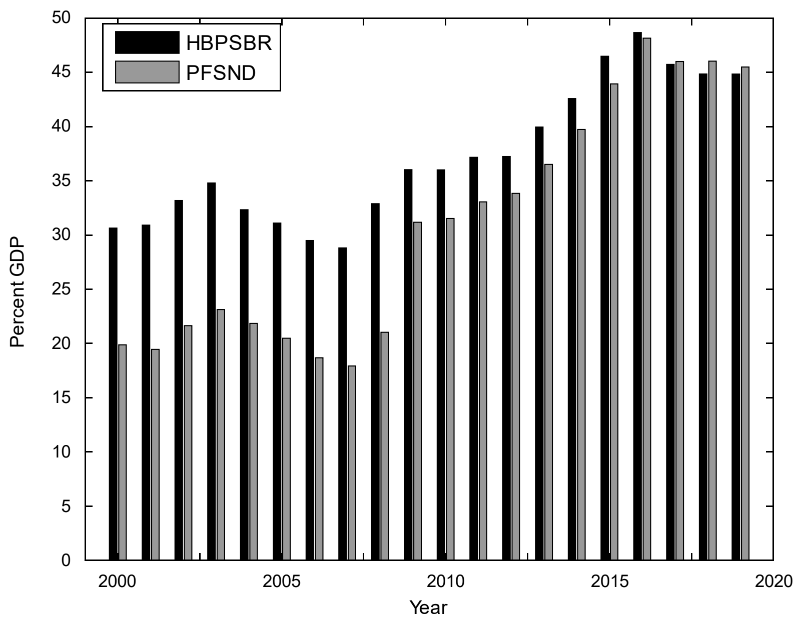

Figure 11: **HBPSBR and PFSND**. Note: HBPSBR stands for the historical balance of the public-sector borrowing requirements over GDP, which is the broadest definition of the government deficit, and PFSND is the public federal-sector net debt over GDP. Source: SHCP (2014), INEGI, and own calculations.

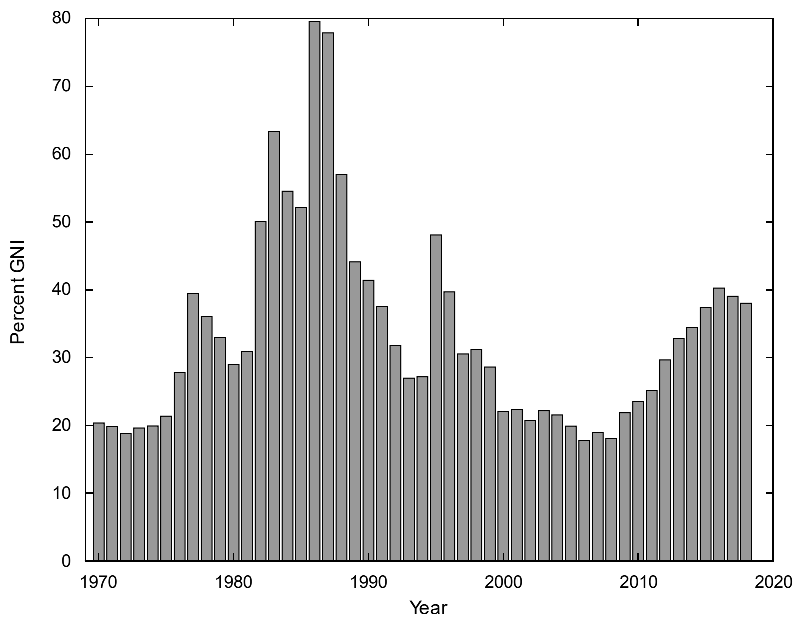

Figure 12: **External Debt as a Percentage of Gross National Income (GNI)**. Source: World Bank.

with these events, the model's inflation expectations increased and the probability of being in a high mean deficit regime started to generally rise again, after a brief reduction in 1976.

In 1982, as financing sources withered due to higher global interest rates and falling oil prices, a deteriorating balance of payments led to capital outflows. The resulting peso devaluation impacted inflation and raised the external debt service. With no access to financing, the government entered a debt moratorium in August. In this context, it nationalized the commercial banks, affecting its credibility severely. As a result, the probability of being in the high mean regime state conspicuously spiked in 1980 (figure 6) to the highest deficit-mean level. The spikes to the highest deficit-mean level were accompanied by spikes in the escape probability, due to the persistence of deficit-mean states. Two such spikes happened during López-Portillo's term, and one during Echeverría's term.

Thus, the fiscal expansion during administrations of Echeverría and López-Portillo resulted in the 1982 balance-of-payments crisis. Our estimations show that the escape-provoking probability initially presented three spikes and then continuously stayed at values significantly higher than zero after 1982 (figure 8).

De la Madrid's presidential term (1982–1988) started with a major stabilization plan. As its key element, it included substantial fiscal retrenchment. This program induced a switch to a lower deficit-mean regime (figures 6 and 10). In spite of such adjustments, inflation kept escalating, although it went down temporarily after a few months. The reason is that seigniorage apparently started to drop after a delay. Even then, the switch to a lower deficit-mean state did not prevent inflation expectations from continuing to increase.

According to our model predictions, equilibrium inflationary expectations as estimated by the SCE were about 10% monthly during this period, and that is why expectations kept increasing. A stronger fiscal retrenchment than implemented was required to stabilize inflation. Toward the end of the De la Madrid administration, the economy returned to a high deficit state due to fiscal pressures caused by external debt payments that were amplified by two large devaluations, in 1986 and then in 1987 (figure 6).

In all fairness, it must be said that in De la Madrid's term the public finances faced several adverse economic shocks. Prominently, in 1985, an earthquake with catastrophic repercussions struck Mexico City. Additionally, in 1986, an oil shock had significant consequences for terms of trade and the fiscal accounts (figure 9).

In 1987, the government enacted an exchange-rate-based (ERB) stabilization program, the Economic Solidarity Pact. One of its main objectives was to deal with inflation persistence. The program included not only a more restrictive fiscal stance (i.e., switching to a lower mean deficit regime) but also a plan to coordinate inflation expectations at a low and stable equilibrium (i.e., to reach a low and stable SCE). Thus, the pact involved income policies (i.e., wage and price controls) and, most importantly, used the exchange-rate to try to anchor the nominal system. Some other modifications took place, such as trade liberalization, deregulation, and divestitures of government companies.

For all the program's careful economic design and implementation, high inflation and inflationary expectations persisted. In fact, inflation reached its maximum in 1987. Indeed, as can be seen in figure 6, the probability of being in a high mean deficit state remained the highest even after the reform. As before, these fiscal reforms either seemed to exert an effect on the deficit-financed seigniorage, only with a delay, or just weren't strong enough since ever-increasing external debts liabilities lent minimal credibility to any fiscal or monetary policy commitment.

Although there were numerous episodes of fiscal retrenchment in the 1980s, there was a significant dependence on seigniorage financing of the deficit, as servicing the very high stock of external public debt remained a key problem. Evidently, the economy had an external public debt overhang (figure 9). There are two highly interlinked key issues when an economy faces a debt overhang predicament: understanding the role of inflation as a resource transfer (risk-sharing) mechanism and the need to renegotiate the external debt. We next discuss both matters.[28]

First, a heavily indebted government needs additional sources of revenue. However, raising taxes, introducing more price controls, or cutting expenditures, in the amount needed to confront a debt overhang problem is basically not possible on most occasions. This difficulty is even more so if, as is usually the case, the country with the debt overhang problem faces acute rollover risk. In this context, inflation acts as a mechanism to transfer resources. This is commonly brought about by exchange-rate devaluations, which affect inflation directly and reduce real wages. Such an effect is feasible given

[28]The existence of inflation as a risk-sharing mechanism could have been relevant for some economies individually in the Eurozone during the 2010 crisis.

the sluggishness with which nominal wages tend to adjust, a reduction in real wages decreases domestic consumption. On the other hand, exchange-rate devaluations lead to a rise in export demand, improving the external accounts. In addition, and perhaps more importantly, inflation dilutes the real value of domestic currency–denominated government nominal debt.[29] Accordingly, inflation serves as a resource transfer mechanism from local residents and nominal debt holders to foreign currency–denominated external debt holders. An inflation tax is used as a resource transfer mechanism when a country tries to avoid defaulting on its external debt. Inflation results from the aggressive devaluations needed to generate the foreign currency resources to service the country's external debt. The process is not a stable one and might result in even higher inflation. It is therefore almost inevitably necessary to renegotiate the external debt.

Second, the Baker Plan (1985), in which Mexico participated in 1986 and 1987, was an attempt at external debt renegotiation. The main idea was that in return for economic reforms, highly indebted economies would obtain access to medium-term loans and to the possibility of rolling over old loans. In principle, with these reforms and fresh credit, high-debt economies would be able to grow their way out of debt. However, for several reasons, this approach was not successful in general (van Wijnbergen, King, and Portes, 1991). Its successor was the Brady Plan, in which Mexico participated in 1989 and which had a better outcome.[30] We will have more to say about this. Note that 1986 and 1987 had the highest external debt over gross national income (GNI) levels (see figure 12). Thereafter, external debt levels decreased.

Again, the inflation maximum was reached in December 1987, and high inflation continued into January 1988. External debt renegotiations started taking place in 1989 under the Brady Plan. It is worth emphasizing that inflation, the external debt, and the regime state probability dynamics are all consistent with the economy exploding toward a very high and stable inflation equilibrium (SCE). In addition, the apparent escape event of 1988 made a fundamental reform unavoidable (figure 8).

During the term of Salinas (1988-1994), another ERB stabilization program was implemented: the Stability and Economic Growth Pact. This

[29] For its "effective" implementation, a government needs an element of surprise. Otherwise, if agents anticipate its actions, nominal variables adjust rapidly.

[30] It takes its name from James Baker, the U.S. secretary of the Treasury at that time. Baker proposed it at the IMF/WB 1985 meetings in South Korea. See Sachs (1989).

included fiscal retrenchment once more as well as wage and price controls akin to those mentioned in Bruno (1989), and it aimed at coordinating inflation expectations toward a low and stable equilibrium. Some structural reforms, including the NAFTA, were also implemented. However and most importantly, the external public debt this time around was successfully renegotiated.

The government owed a substantial portion of its external debt to commercial banks. Banks were unable to sell these loans and thus faced significant concentration risk. Under the plan, an indebted economy would issue Brady bonds and exchange them for such loans.[31] Thus, banks were generally willing to obtain such bonds at a discount and with longer maturities. They were then able to sell their Brady bonds to a third party. The IMF, the World Bank, and the Bank of Japan provided guarantees on their principal and initial coupon payments, leading to lower costs. There was then the possibility for Pareto-improving renegotiations (Sanginés, 1987). Clearly, the external debt renegotiations were pivotal in regaining the stability of inflation expectations.

The Salinas administration implemented further reforms. In step with its trade liberalization, Mexico removed most capital controls. The exchange-rate policy was partially an exception. In November 1991, the authorities set a target zone for the exchange-rate.

Having nationalized the commercial banks in 1982, the government privatized them in 1991–1992. The privatization process raised substantial concerns, such as on the new owners' experience in the sector, the lackluster implementation of international banking standards, and the moral hazard created by the presence of government guarantees for some of the banks' liabilities (Musacchio, 2012). The privatized banks competed intensely for market share. These elements contributed to an outright credit boom. What is more, several commercial banks had funded their market expansions with USD-denominated instruments. Overall, these measures implied a much lower deficit-mean regime, which quickly returned to late-1960s levels (figures 9 and 10).

It is nontrivial to point to a specific cause of the 1994–1995 Mexican crisis. However, the following events surely played a role. The financial liberalization enabled an increase in capital flows. In addition, a large portion of capital was allocated to short-term financial investments. The Federal Re-

[31] Named after the U.S. Treasury secretary at the time, Nicholas Brady.

serve maintained its policy rate at a low level for the initial part of the 1990s. As domestic inflation persisted, local short-term rates were relatively higher, leading to significant portfolio investments from abroad. Importantly, there was a considerable fiscal expansion and a significant misalignment of the real exchange-rate during the latter years of Salinas's term. The undervaluation of the exchange-rate led to a boom in construction and other non-tradeable investments.

Three major political events struck in 1994. A social revolt erupted in January in the state of Chiapas, the leading presidential candidate was assassinated in March while campaigning, and a political leader was murdered in September. On top of these events, the Federal Reserve began raising the target federal funds rate in early 1994. The combination of these events led to considerable outflows and the concomitant loss of international reserves. In response, the government issued dollar-indexed short-term bonds (the so-called *Tesobonos*)–that is, borrowing to defend the exchange-rate (Buiter, 1987). By November, whether the foreign reserves would be sufficient to back such bonds became a substantial concern to foreign investors.

The Zedillo administration (1994–2000) took office in December 1994. After considerable capital outflows, the Bank of Mexico announced it would shift the upper bound of the exchange-rate's target zone by 15%. Capital flows continued pouring out. On December 22, the exchange-rate was left to float and registered a considerable depreciation. The 1994 crisis's repercussions were dramatic. Inflation went up to 52% in 1995 (December) and interest rates to 51% (28-day interbank interest rate), and the GDP fell by 6.3%. In spite of a considerable fiscal retrenchment effort, as the costs associated with the financial sector became apparent, a switch to a higher mean deficit regime took place and lasted for most of 1995. Concomitantly, the escape-provoking probability spiked (figure 8), but by 1997 it had returned to levels close to zero.

There were various elements to the crisis's policy response. The most important was the end of fiscal dominance, as the reforms included the independence of the Bank of Mexico with respect to the fiscal authority. But the response included other elements. By the end of January, a financial support package was announced. It amounted to US$50 billion and involved the participation of the U.S. Treasury, the IMF, the BIS, and private commercial banks. This action likely prevented an insolvency crisis (Sidaoui, 2000). As mentioned, there was a trend toward retrenching fiscally (e.g., Whitt 1996). The government also deployed several programs with the objective of preventing a banking crisis, which entailed providing dollar liquidity to banks, recapitalizing banks that were not satisfying capital requirements, and absorbing bank loans, among other measures. These events are consistent and are captured quite well by the dynamics of the estimated model. The most important is the disappearance of the correlation between fiscal deficits and deficit-financed seigniorage (figure 9). There was also a progressive switching to lower deficit-mean levels in 1997–1998 and an associated stabilization of inflation expectations.

Note that after the initial spike in the deficit-mean level in 1995, it quickly switched to level $m = 3$ and then to progressively lower levels (figure 6). As can be seen in figure 5, levels $m = 3 - 6$ have a unique low-inflation SCE; thus, after reaching $m = 3$, it is feasible to gradually lower the deficit-mean without risk of staying in the domain of attraction of an SCE with very high inflation. The stabilization program could have failed, according to the model, if inflationary expectations were initially in the domain of $\beta_3^*(m)$ and if the deficit-mean was lowered after the initial spike to levels such that an SCE $\beta_3^*(m)$ with very high inflation existed.

Notably, in 1998, there was a sharp oil price drop. Nonetheless, a timely and credible fiscal adjustment avoided further economic deterioration.[32]

Since 1999, the escape-provoking probability has stayed close to zero, and inflation expectations have remained between the two lowest SCEs (figure 7).[33] These results are in line with Ramos-Francia and Torres (2005), who argue that the measures taken after the Tequila Crisis avoided a fiscal dominance situation.

Since 2000–2001, the regime has been mostly in the lowest mean state (figure 6). The trend toward prudent fiscal management continued, and the central bank adopted an inflation-targeting regime. These elements solidified the foundation for a low and stable inflation process. In this regard, Chiquiar, Noriega, and Ramos-Francia (2010) document that inflation transitioned from an I(1) process to an I(0) one around 2001. Interestingly, although the inflation process appeared to become stationary earlier than 2001, it was not until the regime switched to a low mean deficit state that the inflation process (statistically) became so. The probability of being in the lowest-mean deficit state has remained, since then, the highest (figure 4).

[32]The model captures the shock and corresponding adjustment by a small spike in the probability of being in the high mean deficit regime.

[33]Still, one can plausibly think of the government's actions as steps toward the mitigation of inflation expectation instability in the high mean deficit regime.

It is worth noting that there was a temporary switch to the second lowest mean regime during, and after the global financial crisis (GFC), as well as more frequent switchings to the high volatility state.

To summarize this section, we have identified two clearly defined stages during our period of study: first, the fiscal dominance stage before 1995, and second, central bank independence starting in 1995. In figure 9, the model shows a clear association between the government fiscal deficit and the model-implied seigniorage-financed deficit during the first stage. Thus, unconstrained fiscal largesse inevitably resulted in more inflation. During the second stage, fiscal discipline coupled with monetary independence has explained the monetary stability experienced by Mexico during the last twenty years.

It has been also an important finding that, to establish such a close relationship between fiscal dominance, seigniorage, and inflation, we need to use a money-demand function free of Cagan's paradox. We have obtained the simulated seigniorage-finance deficit using the model in SWZ, which uses Cagan's money-demand function and we display it in figure 9. It does not have such a clear association with Mexican fiscal deficit as our model-implied deficit. This result is driven by our variation of the Selden-Latané's money-demand function, and not by the number of states or our volatility function. Indeed, we have verified the relationship between inflation and seigniorage in both models. There is a clear association between these variables in our model and a not so clear relation in the model in SWZ. Thus, the choice of money-demand function is instrumental to establish an even stronger case for the need of fiscal reforms for stabilization efforts, and the maintenance of such reforms for sustained stability.

10 Final Remarks

Historically, Latin American economies have partially financed their fiscal deficits through seigniorage. One common characteristic of the economies in the region is that they have allowed the inflationary tax to take an active role. Mexico has not been the exception in this regard. Several times in the past, it has used the inflationary tax to finance its fiscal deficit; i.e., to close the gap in the government's inter-temporal budget constraints.

As a consequence, the country has had to bear substantial costs in terms of inflation, fiscal imbalances, and indebtedness, and, in some cases, economic crises. In response, the government confronted the associated challenges implementing several adjustment programs, many of which proved initially insufficient. Several factors were crucial for these programs' success. For example, under the presence of debt overhang, the renegotiation of the external debt proved pivotal. Probably the most important factor was the introduction of central bank independence. In the context of the model, such factors made the transition from the high seigniorage-financed deficit-mean regime to the lowest deficit-mean regime durable, making fundamental reforms possible in the model.

The model is able to capture the dynamics of inflation, inflation expectations, and seigniorage-financed fiscal deficits (e.g., see Sargent and Wallace, 1981). In effect, the interaction of the demand for money, the inter-temporal government budget constraint, and the distribution of the fiscal deficits do a good job of characterizing the macroeconomic variables' dynamics. The regimes that are part of the distribution of fiscal deficits enable a better characterization of such dynamics.

The close correlation between actual deficits and model implied deficits is striking up to 1995 in figure 9, depicting a typical fiscal dominance situation. We emphasize that Mexican deficit data was never used in our estimations, and we can think about figure 9 as an out-of-sample prediction. This correlation disappeared in 1995 when central bank independence was implemented, bringing about continuous stability to the economy.

It is worth reemphasizing that our main conclusion hinges on the particular money-demand function we employ. Thus, we consider that the choice of functional form for the money demand equation is crucial to study the relationship between fiscal dominance and inflation. Cagan's paradox obfuscates the true relationship between seigniorage and inflation. On the other hand, since Mexico has not experienced any hyperinflationary event during the period of study, our model contains additional predictions that could help better understand the experiences of countries that have suffered more dramatic inflationary episodes.

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Appendix: The Inflation Likelihood Function.

Inflation shocks follow an independent and identically distributed random variable with probability density

$$Pr_{\pi}(\varepsilon_{\pi}|m_t) = \frac{\exp\left(-[\log(\pi_1^*(m_t)+\varepsilon_{\pi}) - \log(\pi_1^*(m_t))]^2/(2\sigma_{\pi}^2)\right)}{\sqrt{2\pi}[\pi_1^*(m_t)+\varepsilon_{\pi}]\Phi[(\log(\delta)-\log(\pi_1^*(m_t)))/\sigma_{\pi}]},$$

if $-\pi_1^*(m_t) < \varepsilon_{\pi} < \frac{1}{\delta} - \pi_1^*(m_t)$ and 0 in other cases, where $\Phi[\cdot]$ denotes the normal standard cumulative function. On the one hand, the lower bound of the interval $[-\pi_1^*(m_t), 1/\delta - \pi_1^*(m_t)]$ ensures that inflation is positive after a cosmetic reform.

On the other hand, the upper one ensures that inflation is below the upper bound $\delta^(- 1)$, which we have introduced in equation (10). Thus, we denote $s^t = \{s_1, ..., s_t\}$ and $d^t = \{d_1, ..., d_t\}$ for the history of regime states up to period t, and the history of deficits up to t, respectively, we also define

$$\xi_d(s_t, d_{t-1}) = \frac{1}{\sigma_d(s_t, d_{t-1})} = \frac{1}{\sqrt{\sigma_d^2(\varsigma_t) d_{t-1}^{\vartheta}}},$$

$$\xi_\pi = \frac{1}{\sigma_\pi}.$$

Proposition A1. *The conditional likelihood is*

$$\begin{aligned}
&p\big(\pi_t|\pi_{t-1}, s^t, d^{t-1}, \phi\big) \\
&\quad= C_{1t} \frac{|\xi_\pi| \exp\left\{\left(-\frac{\xi_\pi^2}{2}\right)[\log \pi_t - \log \pi_1^*(s_t)]^2\right\}}{\sqrt{2\pi}\Phi\left[|\xi_\pi|\left(-\log\delta - \log\left[\pi_1^*(s_t)\right]\right)\right]\pi_t} \\
&\qquad + C_{2t} \frac{\theta|\xi_d(s_t, d_{t-1})|\lambda(\beta_{t-1})}{\sqrt{2\pi}\left[\lambda(\beta_t)\pi_t - \theta\lambda(\beta_{t-1})\right]\pi_t} \\
&\qquad \times \exp\Big(-\frac{\xi_d^2(s_t, d_{t-1})}{2}\Big\{\log\left[\lambda(\beta_t)\pi_t - \theta\lambda(\beta_{t-1})\right] \\
&\qquad - \log\pi_t - \log\gamma - \log\left[\bar{d}(m_t)\right]\Big\}^2\Big),
\end{aligned}$$

where

$$\begin{aligned}
C_{1t} &= \Big(1 - \Phi\big[|\xi_d(s_t, d_{t-1})| \\
&\qquad \big(\log\left\{\max\left[\left(\lambda(\beta_t) - \delta\theta\lambda(\beta_{t-1})\right)/\gamma, 0\right]\right\} - \log\left[\bar{d}(m_t)\right]\big)\big]\Big), \\
C_{2t} &= \iota\left(\min\left[\frac{\theta\lambda(\beta_{t-1})}{\lambda(\beta_t)}, \frac{1}{\delta}\right] < \pi_t < \frac{1}{\delta}\right).
\end{aligned}$$

Proof: It follows SWZ closely. The likelihood describes what can happen at each $t:$ there is a reform if $\widetilde{\varepsilon}_{dt}(s_t, d_{t-1}) \geq \overline{\omega}_t(s_t, d_{t-1})$. And if there is no reform, the dynamics is driven by the deficit shock jointly with the inflation equilibrium equation. We need to show that

$$\int_0^{1/\delta} p\left(\pi_t|\pi^{t-1}, d^{t-1}, s_t, \phi\right) d\pi_t = 1.$$

Rearranging the definition of $p\left(\pi_t|\pi^{t-1}, d^{t-1}, s_t, \phi\right)$, combining with the definition of $p_d\left(\varepsilon_d|s_t, d_{t-1}\right)$ and $p_\pi\left(\varepsilon_\pi|m_t\right)$, and taking into account that ε_d and ε_π are independent, we get:

$$\begin{aligned}
&p\Big(\pi_t|\pi_{t-1}, s^t, d^{t-1}, \phi\Big) \\
&\quad =\Big(1 - \Phi\big[\xi_d\left(s_t, d_{t-1}\right) \\
&\qquad \Big(\log\left\{\max\left[\left(\lambda\left(\beta_t\right) - \delta\theta\lambda\left(\beta_{t-1}\right)\right)/\gamma, 0\right]\right\} - \log\left[\bar{d}\left(m_t\right)\right]\Big)\ \big]\Big) \\
&\qquad \times \frac{|\xi_\pi| \exp\left\{\left(-\frac{\xi_\pi^2}{2}\right)\left[\log \pi_t - \log \pi_1^*\left(s_t\right)\right]^2\right\}}{\sqrt{2\pi}\Phi\left[|\xi_\pi|\left(-\log\delta - \log\left[\pi_1^*\left(s_t\right)\right]\right)\right]\pi_t} \\
&\qquad + \iota\left(\min\left[\frac{\theta\lambda\left(\beta_{t-1}\right)}{\lambda\left(\beta_t\right)}, \frac{1}{\delta}\right] < \pi_t < \frac{1}{\delta}\right)\ \frac{\theta|\xi_d\left(s_t, d_{t-1}\right)|\lambda\left(\beta_{t-1}\right)}{\sqrt{2\pi}\left[\lambda\left(\beta_t\right)\pi_t - \theta\lambda\left(\beta_{t-1}\right)\right]\pi_t} \\
&\qquad \times \exp\Bigg(- \frac{\xi_d^2\left(s_t, d_{t-1}\right)}{2}\Big\{\log\left[\lambda\left(\beta_t\right)\pi_t - \theta\lambda\left(\beta_{t-1}\right)\right] \\
&\qquad - \log\pi_t - \log\gamma - \log\left[\bar{d}\left(m_t\right)\right]\ \Big\}^2\Bigg) \\
&\quad = \Pr\left[\widetilde{\varepsilon}_{dt}\left(s_t, d_{t-1}\right) \geq \widetilde{\omega}_t\left(s_t, d_{t-1}\right)\right] p_\pi\left(\pi_t - \pi_1^*\left(s_t\right)|s_t\right) \\
&\qquad + \iota\left(\min\left[\frac{\theta\lambda\left(\beta_{t-1}\right)}{\lambda\left(\beta_t\right)}, \frac{1}{\delta}\right] < \pi_t < \frac{1}{\delta}\right) p_d\left(\varepsilon_d|s_t, d_{t-1}\right)\frac{d\varepsilon_{dt}\left(s_t, d_{t-1}\right)}{d\pi_t},
\end{aligned}$$

where we used

$$\begin{aligned}
&p_\pi\big(\pi_t - \pi_1^*\left(s_t\right)|s_t\big) \\
&\qquad\qquad = \frac{|\xi_\pi| \exp\left\{\left(-\frac{\xi_\pi^2}{2}\right)\left[\log \pi_t - \log \pi_1^*\left(s_t\right)\right]^2\right\}}{\sqrt{2\pi}\Phi\left[|\xi_\pi|\left(-\log\delta - \log\left[\pi_1^*\left(s_t\right)\right]\right)\right]\pi_t}, \\
&\Pr\left[\widetilde{\varepsilon}_{dt}\left(s_t, d_{t-1}\right) \geq \overline{\omega}_t\left(s_t, d_{t-1}\right)\right] \\
&\qquad\qquad = 1 - \Phi\Big[\xi_d\left(s_t, d_{t-1}\right)| \\
&\qquad\qquad\quad \Big(\log\left\{\max\left[\left(\lambda\left(\beta_t\right) - \delta\theta\lambda\left(\beta_{t-1}\right)\right)/\gamma, 0\right]\right\} - \log\left[\bar{d}\left(m_t\right)\right]\Big)\Big],
\end{aligned}$$

since $\widetilde{\varepsilon}_{dt} \geq \overline{\omega}_t(s_t, d_{t-1})$ if and only if $\widetilde{\varepsilon}_{dt} \geq \frac{1}{\gamma}(\lambda(\beta_t) - \delta\theta\lambda(\beta_{t-1})) - \bar{d}(m_t)$ if and only if $d_{t-1} \geq \frac{1}{\gamma}(\lambda(\beta_t) - \delta\theta\lambda(\beta_{t-1}))$, and the integration by substitution

$$\begin{aligned} p_d(\varepsilon_d | s_t, d_{t-1}) & \frac{d\varepsilon_{dt}(s_t, d_{t-1})}{d\pi_t} \\ &= \frac{\theta |\xi_d(s_t, d_{t-1})| \lambda(\beta_{t-1})}{\sqrt{2\pi}\left[\lambda(\beta_t)\pi_t - \theta\lambda(\beta_{t-1})\right]\pi_t} \\ &\quad \times \exp\Big(- \frac{\xi_d^2(s_t, d_{t-1})}{2} \Big\{ \log\left[\lambda(\beta_t)\pi_t - \theta\lambda(\beta_{t-1})\right] \\ &\quad - \log \pi_t - \log \gamma - \log\left[\bar{d}(m_t)\right] \Big\}^2 \Big), \end{aligned}$$

to obtain an integral with respect to inflation using the equilibrium inflation equation

$$\gamma d_t(s_t, d_{t-1}) = \frac{\lambda(\beta_t)\pi_t - \theta\lambda(\beta_{t-1})}{\pi_t}.$$

We now recall that

$$\int_0^{1/\delta} p_\pi(\pi_t - \pi_1^*(s_t) | s_t) \, d\pi_t = 1$$

and that

$$\begin{aligned} \int_0^{1/\delta} \iota\Big(\min\left[\frac{\theta\lambda(\beta_{t-1})}{\lambda(\beta_t)}, \frac{1}{\delta}\right] < \pi_t < \frac{1}{\delta}\Big) p_d(\varepsilon_d | s_t, d_{t-1}) \frac{d\varepsilon_{dt}(s_t, d_{t-1})}{d\pi_t} d\pi_t & \\ = \int_{L_t}^{1/\delta} p_d(\varepsilon_d | s_t, d_{t-1}) \frac{d\varepsilon_{dt}(s_t, d_{t-1})}{d\pi_t} d\pi_t & \\ = \int_{-\bar{d}(s_t)}^{\overline{\omega}_t(s_t, d_{t-1})} p_d(\varepsilon_d | s_t, d_{t-1}) \, d\varepsilon_{dt}(s_t, d_{t-1}) & \\ = \Pr\left[\widetilde{\varepsilon}_{dt}(s_t, d_{t-1}) < \widetilde{\omega}_t(s_t, d_{t-1})\right] & \end{aligned}$$

where we used the integration by substitution of deficit by inflation and where

$$L_t = \min\left[\frac{\theta\lambda(\beta_{t-1})}{\lambda(\beta_t)}, \frac{1}{\delta}\right];$$

then, collecting terms:

$$\begin{aligned}
&\int_0^{1/\delta} p\left(\pi_t|\pi^{t-1}, d^{t-1}, s_t, \phi\right) d\pi_t \\
&= \int_0^{1/\delta} \Bigg[\Pr\left[\widetilde{\varepsilon}_{dt}\left(s_t, d_{t-1}\right) \geq \widetilde{\omega}_t\left(s_t, d_{t-1}\right)\right] p_\pi\left(\pi_t - \pi_1^*\left(s_t\right)|s_t\right) \\
&\quad + \iota\left(\min\left[\frac{\theta\lambda\left(\beta_{t-1}\right)}{\lambda\left(\beta_t\right)}, \frac{1}{\delta}\right] < \pi_t < \frac{1}{\delta}\right) p_d\left(\varepsilon_d|s_t, d_{t-1}\right) \frac{d\varepsilon_{dt}\left(s_t, d_{t-1}\right)}{d\pi_t}\Bigg] d\pi_t \\
&= \Big[\Pr\left[\widetilde{\varepsilon}_{dt}\left(s_t, d_{t-1}\right) \geq \widetilde{\omega}_t\left(s_t, d_{t-1}\right)\right] + \Pr\left[\widetilde{\varepsilon}_{dt}\left(s_t, d_{t-1}\right) < \widetilde{\omega}_t\left(s_t, d_{t-1}\right)\right] \Big] \\
&= 1. \ \textit{Q.E.D.}
\end{aligned}$$

Integrating out s^T , we have the likelihood of interest

$$\begin{aligned}
p\left(\pi^T|\phi\ \right) &= \prod_{t=1}^{T} p\left(\pi_t|\pi^{t-1}, d^{t-1}, \phi\right) \\
&= \prod_{t=1}^{T} p\left(\pi_t|s_t, \pi^{t-1}, d^{t-1}, \phi\right) \Pr\left(s_t|\pi^{t-1}, d^{t-1}, \phi\right)
\end{aligned}$$

where

$$\Pr\left(s_t|\pi^{t-1}, d^{t-1}, \phi\right) = \sum_{s_{t-1}=1}^{h} \Pr\left(s_t|s_{t-1}, Q_s\right) \Pr\left(s_{t-1}|\pi^{t-1}, d^{t-1}, \phi\right).$$

As in Sims, Waggoner and Zha (2008) and SWZ, the probability of having observed a state, $\Pr\left(s_{t-1}|\pi^{t-1}, d^{t-1}, \phi\right)$ can be updated recursively starting with the assumption that

$$\Pr\left(s_0|\pi^0, d^0, \phi\right) = 1/h.$$

That is, at the beginning, the econometrician does not know in which state he is, so he assigns the same probability to each state. Thus, using the recursion process, we have that:

$$\Pr\left(s_t|\pi^t, d^{t-1}, \phi\right) = \frac{p\left(\pi_t|\pi^{t-1}, d^{t-1},\ s_t,\ \phi\right) \Pr\left(s_t|\pi^{t-1}, d^{t-1}, \phi\right)}{\sum_{s_t=1}^{h} \left[p\left(\pi_t|\pi^{t-1}, d^{t-1}, s_t,\ \phi\right) \Pr\left(s_t|\pi^{t-1}, d^{t-1}, \phi\right)\right]}.$$